Exercise AP-211

Present value of a delayed infinite stream

The Economic Skills Project

1 Problem

Problem

What is the present value in year 0 of an infinite stream of \$1500 payments starting in year 11 when the interest rate is 5%?

2 Answer

Answer

Here's the solution:

- \$18,417

3 Method

Solution method

Here's one approach:

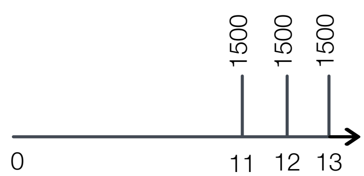
1. Draw the cash flow diagram.
2. Use the delayed infinite stream formula.
3. Check using fundamental principles.

4 Solution

4.1 Step 1

Draw the cash flow diagram

Here's how it looks:



4.2 Step 2

Use the delayed infinite stream formula

The present value of an infinite stream of identical payments F starting at time $T+1$ when the interest rate is r is given by:

$$PV = \frac{F/r}{(1+r)^T}$$

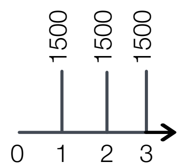
Since the first payment arrives in year 11, $T+1 = 11$ so $T = 10$. Filling in the other numbers and calculating gives:

$$PV = \frac{\$1500/0.05}{1.05^{10}} = \$18,417$$

4.3 Step 3

Checking using fundamentals

It's possible to check the answer using fundamental principles of present value calculations. First, notice that someone in period 10 would see the payments as an infinite stream starting one year in the future:

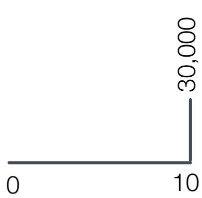


Value in year 10?

To that person in year 10, the present value of the payments could be computed using the infinite stream formula:

$$PV = \frac{F}{r} = \frac{\$1500}{0.05} = \$30,000$$

That means that the original stream is equivalent to receiving a single payment of $\$30,000$ in year 10:



Value in year 0?

To complete the problem it's necessary to step from year 10 back to year 0. The present value of $30,000 in year 10 can be calculated using the formula for a single payment:

$$PV = \frac{F_t}{(1+r)^T}$$

In this case $t = 10$, $r = 0.05$ and $F_t = \$30,000$ so the final present value is:

$$PV = \frac{\$30,000}{1.05^{10}} = \$18,417$$

Everything checks - done!