Exercise AP-231

Present value of a delayed finite stream

The Economic Skills Project

1 Problem

Problem

What is the present value in year 0 of a stream of $5 million payments starting in year 6 and ending in year 25 when the interest rate is 7%?

2 Answer

Answer

Here's the solution:

- $37.8 million

3 Method

Solution method

Here's one approach:

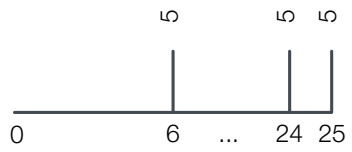
1. Draw the original cash flow diagram.
2. Draw the cash flow from year 5's perspective.
3. Use the finite stream formula in year 5 to find a equivalent lump sum.
4. Take the present value of the year-5 lump sum.

4 Solution

4.1 Step 1

Draw the original cash flow diagram

Here's how it looks:



4.2 Step 2

Draw the cash flow from year 5's perspective

Here's how it looks:

4.3 Step 3

Use the finite stream formula in year 5

The present value of an finite stream of identical payments F starting at time 1 and ending at T when the interest rate is r is given by:

- $\text{PV} = \frac{\text{F}}{\text{r}}\left(1 - \frac{1}{(1+\text{r})^{\text{T}}}\right)$

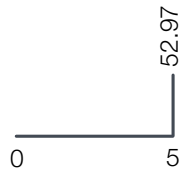
From year 5's perspective, the payments occur from 1 year (year 6 - year5) to 20 years (year 25 - year 5) in the future. Thus, the present value in year 5 can be computed from the equation above using $\text{T} = 20$. Filling in the other numbers and calculating gives:

- $\text{PV} = \frac{\$5\text{M}}{0.07}\left(1 - \frac{1}{(1+0.07)^{20}}\right) = \$52.97\text{M}$

4.4 Step 4

Take the present value of the year-5 lump sum

Step 3 shows that the original stream is equivalent to a lump sum payment of $52.97 million in year 5. Thus, the original stream is eqivalent to the following cash flow, in millions of dollars:



Apply the single payment formula

The last step applies the basic present value formula to that payment:

- $\text{PV} = \frac{\text{F}}{(1+\text{r})^{\text{T}}}$

Inserting the values:

- $\text{PV} = \frac{\$52.97\text{M}}{(1+0.07)^{5}} = \$37.8\text{M}$

Done!