Exercise AU-201

Highway planning with uncertain population growth

The Economic Skills Project

1 Problem

Problem

A city is considering two upgrades to its highway system: H and L. H would have high capacity and would cost $100 million. L would be low capacity system and would cost $25 million.

The benefits of H depend on future growth of the city. There is a 40% chance the citys growth will be fast (F) and the benefits of H would be $200 million. However, there is a 60% chance growth will be slow (S) and benefits of H would be $50 million.

The benefits of L will be $30 million for either growth rate.

Finally, the city could pay a consultant $5 million to determine the citys likely future growth rate.

Problem, continued

Please determine the citys best course of action and give its expected value. To keep things simple you may assume that all the numbers given above are present values (so no additional PV calculations are needed) and that the consultant could carry out the study immediately and would be correct in its prediction.

2 Answer

Answer

The city's best option is to hire the consultant. The expected NPV is $38M. The second best option would be to build the high capacity system without the consultant. However, that has an expected NPV of only $10M.

3 Method

Solution method

Here's one approach:

1. Draw the decision tree.
2. Evaluate the tree from right to left.

4 Solution

4.1 Step 1

Draw the decision tree

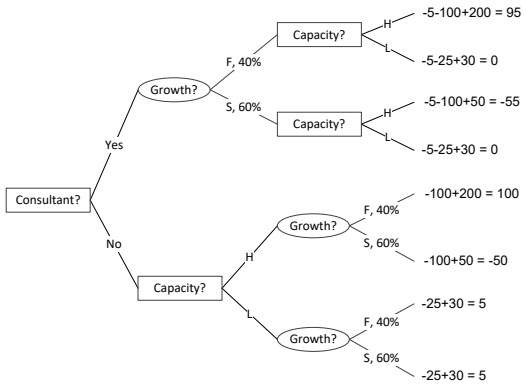
The tree is shown on the next page.

The parts of each payoff are shown in the order they occur along the tree. For example, the top payoff is shown as:

$$-5 - 100 + 200 = 95$$

This shows that hiring the consultant occurs first and cost \$5M, then building the high capacity system costs \$100M, and finally the city's benefit is \$200M since growth was fast along that path. The overall payoff is \$95M.

Draw the decision tree



4.2 Step 2

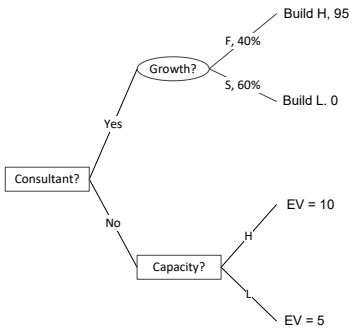
Evalute the tree from right to left

Evaluating the tiers at the far right side of the diagram goes as follows, from top to bottom:

- Choice node; best option is to build H for a payoff of $95.
- Choice node; best option is to build L for a net gain of $0.
- Chance node; EV is $10M.
- Chance node; EV is $5.

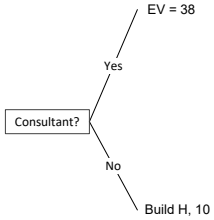
Evaluate the tree from right to left

Revising the tree:



Evalute the tree from right to left, continued

Evaluating the chance node at the top gives an EV of $38M. The best action at the choice node at the bottom is to build H, which has an EV of $10. Updating the tree accordingly:



Evaluate the tree from right to left, continued

Finally, the best option at the initial decision is to hire the consultant for an EV of $38. The next best alternative is to build H without the consultant, which has an expected NPV of only $10M.

Done!