

Exercise CI-101

Saving by an individual with perfect complements preferences

The Economic Skills Project

1 Problem

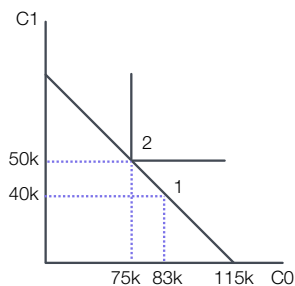
Problem

An individual is concerned about her consumption in two periods, 0 and 1. In period 0 her income will be \$83,000 and in period 1 it will be \$40,000. She would like to have exactly 3 units of consumption in 0 for every 2 units of consumption in 1. She can borrow or save at an interest rate of 25%. How much does she borrow or save in 0? Show her equilibrium in a diagram.

2 Answer

Answer

She saves \$8000 and moves from her endowment, point 1 in the figure, to the equilibrium shown by point 2.



3 Method

Solution method

Here's one approach:

1. Write out the intertemporal budget constraint.
2. Solve for an equation linking her preferred C_0 and C_1 .
3. Use the result from 2 and the budget constraint to solve for C_1 .
4. Solve for C_0 and compare it to I_0 to determine her saving.
5. Draw the diagram.
6. Check the result.

4 Solution

4.1 Step 1

Write out the intertemporal budget constraint

The general budget constraint for a two-period intertemporal choice problem is:

$$C_0 + \frac{C_1}{1+r} = I_0 + \frac{I_1}{1+r}$$

Inserting r , I_0 , and I_1 :

$$C_0 + \frac{C_1}{1.25} = \$83,000 + \frac{\$40,000}{1.25}$$

Calculating the right hand side (the present value of her income) gives:

$$C_0 + \frac{C_1}{1.25} = \$115,000$$

4.2 Step 2

Solve for an equation linking C_0 and C_1

Since she wants exactly 3 units of consumption in period 0 for every 2 units in period 1, her preferred ratio of consumption is:

$$\frac{C_0}{C_1} = \frac{3}{2}$$

Solving this for C_0 in terms of C_1 :

$$C_0 = 1.5C_1$$

4.3 Step 3

Solve for C_1

Her intertemporal budget constraint was:

$$C_0 + \frac{C_1}{1.25} = \$115,000$$

Using her preference relationship to eliminate C_0 :

$$1.5C_1 + \frac{C_1}{1.25} = \$115,000$$

Collecting terms in C_1 :

$$2.3C_1 = \$115,000$$

Solving for C_1 :

$$C_1 = \$50,000$$

4.4 Step 4

Solve for C_0 and her saving

Use her preference relationship to determine C_0 from C_1 :

$$C_0 = 1.5C_1 = \$75,000$$

Her saving will be her income in period 0 less her consumption:

$$S = I_0 - C_0$$

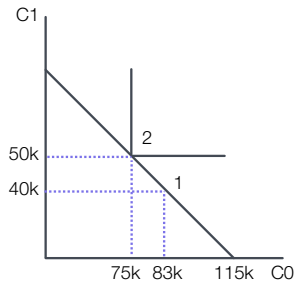
Filling in the numbers:

$$S = \$83,000 - \$75,000 = \$8,000$$

4.5 Step 5

Draw the diagram

Her endowment is point 1: $C_0 = \$83k$ and $C_1 = \$40k$. Her preferred consumption bundle is point 2: $C_0 = \$75k$ and $C_1 = \$50k$. Since she has perfect complements preferences, her indifference curves are right angles.



4.6 Step 6

Check the result

One way to check the result is to calculate the present value of her consumption bundle:

$$C_0 + \frac{C_1}{1.25} = \$75,000 + \frac{\$50,000}{1.25} = \$115,000$$

Since that equals the present value of her income, it passes the check.

An alternative check is to compute her total future income, including from saving, to make sure it matches her consumption. The value of her savings in 1 is $\$8,000 \cdot 1.25 = \$10,000$. Added to I_1 , she would have $\$50,000$ to spend in 1. Since that exactly matches C_1 , the results pass the second test as well.

Everything checks - done!