Exercise CW-201

Compensating variation for a tax

The Economic Skills Project

1 Problem

Problem

A household buys two goods, X and Y, and its preferences can be represented by the utility function below. Also shown are the household's demand equations and its expenditure function.

$$U = X^{0.5} + Y^{0.5}, \quad X = \frac{M \cdot P_y}{P_X \cdot (P_X + P_Y)}, \quad Y = \frac{M \cdot P_X}{P_Y \cdot (P_X + P_Y)}, \quad M = \frac{U^2 \cdot P_X \cdot P_Y}{P_X + P_Y,}$$

Initially, the household's income is $M_1 = \$3600$ and the prices of the goods are $P_{X1} = \$8$ and $P_{Y1} = \$8$. The government is considering a tax on X that would raise its price to $P_{X2} = \$12$. What is the compensating variation for the policy?

2 Answer

Answer

Here's the solution:

- \$720. Since the CV is positive, the household is worse off.

3 Method

Solution method

Here's one approach:

1. Use the demand equations to compute X_1 and Y_1.
2. Use the utility function to compute U_1.
3. Use the expenditure function to compute M_3.
4. Subtract M_2, from M_3 to obtain the CV.

4 Solution

4.1 Step 1

Use the demand equations to compute X_1 and Y_1

Inserting the initial values of M_1, P_{X1}, and P_{Y1} into the demands gives:

$$X_1 = \frac{\$3200 \cdot \$8}{\$8 \cdot (\$8 + \$8)} = 225$$

$$Y_1 = \frac{\$3200 \cdot \$8}{\$8 \cdot (\$8 + \$8)} = 225$$

4.2 Step 2

Use the utility function to compute U_1

Using X_1 and Y_1 to compute U_1:

$$U_1 = 225^{0.5} + 225^{0.5} = 15 + 15 = 30$$

4.3 Step 3

Use the expenditure function to compute M_3

Inserting U_1 and P_{X2} and P_{Y2} into the expenditure function gives M_3, the expenditure needed to get the original utility at the new prices. The new price of X was given by the policy and the new price of Y was unchanged from the original value. Thus:

$$M_3 = \frac{30^2 \cdot (\$12 \cdot \$8)}{\$12 + \$8} = \$4320$$

4.4 Step 4

Subtract M_2 from M_3 to get the CV

The amount of income needed to get the household back to the original utility is:

$$CV = M_3 - M_2 = \$4320 - \$3600 = \$720$$

To make the household as well off as it was initially, it would need to be given \$720. In other words, the policy makes the household worse off by \$720.

Done!