Exercise CW-202

Compensating variation for a multipart policy

The Economic Skills Project

1 Problem

Problem

A household buys two goods, X and Y, and its preferences can be represented by the utility function below. Also shown are the household's demand equations and its expenditure function.

$$U = (X - 50)^{0.5}(Y + 50)^{0.5}, \quad M = 2UP_X^{0.5}P_Y^{0.5} + 50P_X - 50P_Y$$

$$X = 50 + \frac{0.5(M - 50P_X + 50P_Y)}{P_X}, \quad Y = -50 + \frac{0.5(M - 50P_X + 50P_Y)}{P_Y}$$

Continued...

Problem, continued

Initially, the household's income is $M_1 = \$1000$ and the prices of the goods are $P_{X1} = \$5$ and $P_{Y1} = \$5$. The government is considering a policy that would raise to P_X by \$1 to $P_{X2} = \$6$, lower P_Y by \$1 to $P_{Y2} = \$4$, and also provide a \$50 lump sum transfer to the household. What is the compensating variation for the policy?

2 Answer

Answer

Here's the solution:

- \$30. Since the CV is positive, the household is worse off.

3 Method

Solution method

Here's one approach:

1. Use the demand equations to compute X_1 and Y_1.
2. Use the utility function to compute U_1.
3. Use the expenditure function to compute M_3.
4. Subtract M_2, from M_3 to obtain the CV.

4 Solution

4.1 Step 1

Use the demand equations to compute X_1 and Y_1

Inserting the initial values of M_1, P_{X1}, and P_{Y1} into the demands gives:

$$X_1 = 50 + \frac{0.5(\$1000 - 50 \cdot \$5 + 50 \cdot \$5)}{\$5} = 150$$

$$Y_1 = -50 + \frac{0.5(\$1000 - 50 \cdot \$5 + 50 \cdot \$5)}{\$5} = 50$$

4.2 Step 2

Use the utility function to compute U_1

Using X_1 and Y_1 to compute U_1:

$$U_1 = (150 - 50)^{0.5}(50 + 50)^{0.5} = 100$$

4.3 Step 3

Use the expenditure function to compute M_3

Inserting U_1 and P_{X2} and P_{Y2} into the expenditure function gives M_3, the expenditure needed to get the original utility at the new prices:

$$M_3 = 2U_1 P_{X2}^{0.5} P_{Y2}^{0.5} + 50P_{X2} - 50P_{Y2}$$

Putting in the numbers:

$$M_3 = 2 \cdot 100 \cdot (\$6)^{0.5}(\$4)^{0.5} + 50 \cdot \$6 - 50 \cdot \$4$$

Calculating it out:

$$M_3 = \$1080$$

4.4 Step 4

Subtract M_2 from M_3 to get the CV

Including the lump sum transfer gives $M_2 = \$1000 + \$50 = \$1050$. The amount of income needed to get the household back to the original utility is:

$$CV = M_3 - M_2 = \$1080 - \$1050 = \$30$$

To make the household as well off as it was initially, it would need to be given $30. In other words, the policy makes the household worse off by $30.

Done!

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