Exercise MD-151

Deriving a market demand curve with heterogeneous buyers

The Economic Skills Project

1 Problem

Problem

A market has two types of buyers, A and B. There are 10 type-A buyers and 20 type-B buyers. An individual i of each type has a willingness to pay for the good given by the corresponding equation below.

Type A individual $WTP_i^A = 100 - \frac{1}{2}Q_i^A$

Type B individual $WTP_i^B = 100 - \frac{1}{4}Q_i^B$

What is the market demand $Q_M(P^d)$ where P^d is the price faced by buyers?

2 Answer

Answer

Here's the solution:

- $Q_M = 10,000 - 100P^d$

3 Method

Solution method

Here's one approach:

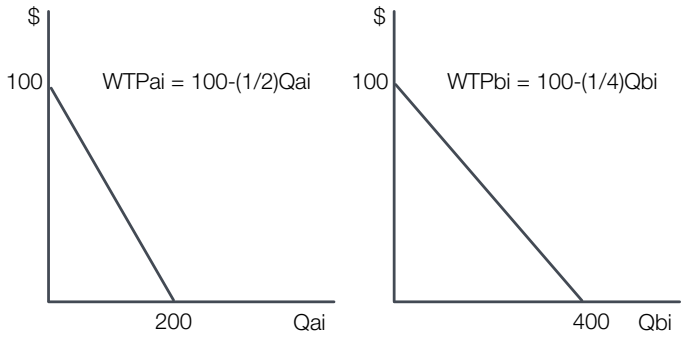
1. Draw graphs of each WTP curve
2. Use the choice rule for a type-A buyer
3. Solve for individual demand Q_i^A
4. Repeat to get individual demand Q_i^B
5. Sum over all the individuals
6. Check the result

4 Solution

4.1 Step 1

Draw the WTP graphs

Here's how they look:



4.2 Step 2

Use the choice rule for type-A buyer

Buyer i of type A facing price P^d chooses Q_i^A where:

- $WTP_i^A = P^d$

Thus for a type-A buyer we have two equations and three variables:

1. $WTP_i^A = 100 - \frac{1}{2}Q_i^A$
2. $WTP_i^A = P^d$

By combining the equations we can derive a single equation giving Q_i^A in terms of P^d.

4.3 Step 3

Solving for individual demand Q_i^A

Use the decision rule (equation 2) to eliminate WTP_i^A from the WTP equation (equation 1) and then solve for Q_i^A:

- $P^d = WTP_i^A = 100 - \frac{1}{2}Q_i^A$
- $P^d = 100 - \frac{1}{2}Q_i^A$
- $\frac{1}{2}Q_i^A = 100 - P^d$
- $Q_i^A = 2\left(100 - P^d\right)$
- $Q_i^A = 200 - 2P^d$

4.4 Step 4

Solving for individual demand Q_i^B

Follow the same set of steps for a person of type B:

- $P^d = WTP_i^B = 100 - \frac{1}{4}Q_i^B$
- $P^d = 100 - \frac{1}{4}Q_i^B$
- $\frac{1}{4}Q_i^B = 100 - P^d$
- $Q_i^B = 4\left(100 - P^d\right)$
- $Q_i^B = 400 - 4P^d$

4.5 Step 5

Summing over individuals

The market demand, Q_M, is the sum of the individual demands taking into account the number of people of each type. If there are N_A people of type A and N_B people of type B, it is:

- $Q_M = \sum_{i=1}^{N_A} Q_i^A + \sum_{i=1}^{N_B} Q_i^B$
- $Q_M = N_A Q_i^A + N_B Q_i^B$

Filling in the given numbers of individuals and the demands derived above:

- $Q_M = 10\left(200 - 2P^d\right) + 20(400 - 4P^d)$
- $Q_M = 2000 - 20P^d + 8000 - 80P^d$
- $Q_M = 10,000 - 100P^d$

4.6 Step 6

Checking the result

The intercepts of the market demand curve should be consistent with the original WTP curves. We'll check the P intercept first.

For both buyers, Q is 0 when $P^d = 100$ so the market demand should have $Q_M = 0$ when $P^d = 100$. Checking:

$P^d = 100$**:** $Q_M = 10,000 - 100(100) = 0$

So far, so good. Next we'll check the Q intercept.

Checking the result, continued

The Q intercepts of the individual WTP curves were $Q_i^A = 200$ and $Q_i^B = 400$. Since there are 10 type-A people and 20 type-B people, the total demand should be:

- $Q_M = 10 \cdot 200 + 20 \cdot 400 = 10,000$

Checking:

- $P^d = 0$: $Q_M = 10,000 - 100(0) = 10,000$

Everything checks - done!