A Simple Numerical Example of the Ramsey Model

Peter J. Wilcoxen
The Maxwell School, Syracuse University

This note describes the theory underlying the numerical version of the Ramsey Model implemented in the accompanying Ox files, Gempack and Excel files. For more information, see http://wilcoxen.cp.maxwell.syr.edu/pages/ramsey/.

The household chooses the path of consumption to maximize the following intertemporal utility function, in which the rate of time preference is ρ and the intertemporal elasticity of substitution is 1 due to the use of $\ln(C)$ for the intratemporal utility index:

$$U = \int_0^\infty \ln C(t) e^{-\rho t} dt \tag{1}$$

There is a single good produced using only capital according to a diminishing returns production function of the form: $f(k) = k^\phi$, where $\phi < 1$. The government collects T units of the good as an in-kind tax and the remainder, Q, is split between consumption, C, and investment, I. Thus, the household is subject to the following constraints:

$$Q = k^\phi - T \tag{2}$$

$$Q = C + I \tag{3}$$

Finally, the capital stock evolves according to the following two conditions:

$$\dot{k} = I - \delta k \tag{4}$$

$$k(0) = k_0 \tag{5}$$

Combining (2), (3) and (4) produces the following overall constraint on the household's optimization:

$$\dot{k} = k^\phi - T - C - \delta k \tag{6}$$

Setting up and solving the household's problem produces the following first-order conditions:

$$C = \frac{1}{\lambda} \tag{7}$$

$$\dot{\lambda} = \left(\rho + \delta - \phi k^{\phi-1}\right)\lambda \tag{8}$$

$$\dot{k} = k^{\phi} - T - \frac{1}{\lambda} - \delta k \tag{9}$$

The last two, (8) and (9), are the model's equations of motion. The steady state can be shown to be:

$$k_{ss} = \left(\frac{\rho + \delta}{\phi}\right)^{1/(\phi-1)}$$

$$\lambda_{ss} = \frac{1}{k_{ss}^{\phi} - \delta k_{ss} - T}$$

$$C_{ss} = k_{ss}^{\phi} - \delta k_{ss} - T$$