Peter J. Wilcoxen | Department of Public Administration
PPA 723, Managerial Economics | The Maxwell School, Syracuse University

Solution to Exam 1
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Here are notes on the solution. Some of the graphs may be omitted and the explanations are a bit terse. If you have any questions, please don't hesitate to stop by during office hours or the lab session to talk over things in detail.

Part 1

1(a) Finding the initial equilibrium

The first step is to solving for the individual demand curves for people of each type. For type A, the curve is given:

Qa = 200

For type B, it is found as follows:

W2Pb = P

40 – 0.2*Qb = P

0.2*Qb = 40 – P

Qb = (40 – P)/0.2

Qb = 200 – 5*P

The market demand (Q) is the sum over all the buyers:

Qd = 5*Qa + 10*Qb

Qd = 5*(200) + 10*(200 – 5*P)

Qd = 1000 + 2000 – 50*P

Qd = 3000 – 50*P

On the supply side (using Qs for clarity):

W2A = P

Qs/100 = P

$Qs = 100*P$

Finding the equilibrium:

$Qd = Qs$

$3000 – 50*P = 100*P$

$3000 = 150*P$

$P = 20$

$Qs = 100*P = 100*20 = 2000$

Checking: $Qd = 3000 – 50*20 = 3000 – 1000 = 2000$

Consumption by a person of each type:

$Qa = 200$

$Qb = 200 – 5*P = 200 – 5*20 = 100$

Checking: $Qd = 5*200 + 10*100 = 1000 + 1000 = 2000$

1(b) Finding the equilibrium with a $12 tax

The tax introduces a gap between what the buyer pays (Pb) and what the seller gets to keep (Ps):

$Pb = Ps + tax = Ps + \$12$

Rearranging the demand curve to find Pb for any given Q:

$Qd = 3000 – 50*Pb$

$50*Pb = 3000 – Qd$

$Pb = (3000 – Qd)/50$

The supply curve is already in the right form:

$Ps = Qs/100$

Inserting Pb and Ps into the equation with the tax, and setting Qd=Qs=Q:

$(3000 – Q)/50 = Q/100 + 12$

$3000 – Q = Q/2 + 12*50 = Q/2 + 600$

$2400 = (3/2)*Q$

$Q = 1600$

$Ps = 1600/100 = 16$

$Pb = (3000 – 1600)/50 = 1400/50 = 28$

Checking: $Ps + 12 = 16 + 12 = 28 = Pb$

Consumption by each type:

$Qa = 200$

$Qb = 200 – 5*Pb = 200 – 5*28 = 200 – 140 = 60$

Checking: $5*200 + 10*60 = 1000 + 600 = 1600$

1(c) Revenue and surplus

Overall effects:

Revenue = 12*1600 = 19,200

$\Delta CS = -((28-20)*1600 + 0.5*(28-20)*400) = -(12,800 + 1,600) = -14,400$

$\Delta PS = -((20-16)*1600 + 0.5*(20-16)*400) = -(6,400 + 800) = -7,200$

DWL = lost surplus less gain in revenue

DWL = (14,400+7,200) – 19,200

DWL = 2,400

Change in CS for each type:

$\Delta CSa = - (28-20)*200 = - 1,600$

$\Delta CSb = (28-20)*60 + 0.5*(28-20)*(100-60) = 480 + 160 = 640$

Type-A consumers are hurt more because they consumed more of the good to begin with, and because their demand is inelastic. Type-B consumers avoid some of the tax because their consumption of the good drops.

Part 2

2(a) Effect of the tax

Since the W2A curve is perfectly elastic at $1, the supply curve with the tax will be horizontal at Ps = W2A + $0.2 = $1.20. The new price will thus be $1.20. To find the new quantity, use the elasticity of demand:

$\eta = \%\Delta Q / \%\Delta P$

$\%\Delta Q = \eta * \%\Delta P$

$\%\Delta P = (1.20 - 1)/1 = 0.2 = 20\%$

$\%\Delta Q = -0.5*20\% = -10\% = -0.1$

$\Delta Q = -0.1*1M = -100{,}000$

$Q = 1M - 100{,}000 = 900{,}000$

Effects on revenue and surplus:

Revenue = $0.2*900,000 = $180,000

$\Delta CS = -(0.2*900{,}000 + 0.5*0.2*(1M-900{,}000)) = -(180K + 10K) = -190{,}000$

$\Delta PS = 0$

DWL = 190,000 – 180,000 = 10,000

Checking: DWL = 0.5*0.2*100K = $10,000

2(b) Effect of the subsidy

Because the W2A curve is perfectly elastic at $10, the initial price will be $10 as well. The $2 subsidy, therefore, will lower the buyer price to $8:

Ps = Pb + subsidy

Pb = Ps – subsidy = $10 - $2 = $8

The change in Q can be found using the elasticity:

$\eta = \%\Delta Q / \%\Delta P$

$\%\Delta Q = \eta * \%\Delta P$

$\%\Delta P = (8 - 10)/10 = -0.2 = -20\%$

$\%\Delta Q = -1*(-20\%) = 20\% = 0.2$

$\Delta Q = 0.2*100K = 20{,}000$

$Q = 100K + 20{,}000 = 120{,}000$

Computing the amount needed for the subsidy:

Subsidy cost = \$2*120,000 = \$240,000

The cost of the subsidy is substantially higher than the revenue raised by the tax on good A. The policy will create a deficit of \$240,000 - \$180,000 = \$60,000.

Part 3

The market diagram is shown below. From the perspective of employers, the \$7 minimum is a 40% increase in the wage: (7-5)/5 = 0.4. They will reduce the number of hours they purchase to Q2, which can be computed using the demand elasticity.

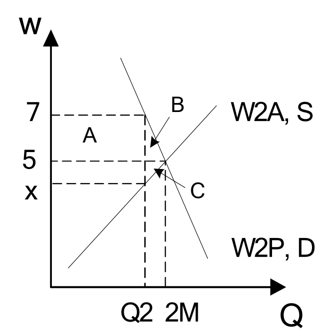
$\eta = \%\Delta Q / \%\Delta P$

$\%\Delta Q = \eta * \%\Delta P$

$\%\Delta Q = -0.5*(40\%) = -20\%$

$\Delta Q = -0.2*2$ million = -400,000 hours

Q2 = 2 million – 400,000 = 1.6 million hours



The effect on employers is the reduction in consumer surplus they receive. In the diagram, it's areas A+B. Computing it:

$\Delta CS = -(A+B)$

$\Delta CS = -(\$2*1.6\text{ M} + 0.5*\$2*400\text{ K}) = -(\$3.2\text{ M} + \$400\text{ K}) = -\$3.6\text{ M}$

The effect on employees is mixed. On one hand, they work 400,000 fewer hours but on the other hand, those working are paid more. As a group, they gain some producer surplus from the higher wage (area A) but lose some due to the cut in hours (area C). Area A was already computed above and is \$3.2 M. To compute area C, it's necessary to determine "x": the W2A at Q2. That can be done using the supply elasticity as follows:

$\eta s = \%\Delta Q / \%\Delta P$

$\%\Delta Q = -20\%$ (from above)

$-1 = 20\% / \%\Delta P$

$\%\Delta P = -20\%$

$\Delta P = -0.2 * \$5 = -\$1$

x = \$5 - \$1 = \$4

C = 0.5*(\$5-\$4)*400,000 = \$200 K

ΔPS = \$3.2 M - \$200 K = \$3 M

Calculating deadweight loss:

DWL = cost to employers – benefits to employees

DWL = \$3.6 M - \$3M = \$600 K

Checking: DWL = 0.5*(\$7-\$4)*400 K = \$600 K