

Solution to Exam 2
Fall 2007

Here are notes on the solution. Some of the graphs may be omitted and the explanations are a bit terse. If you have any questions, please don't hesitate to stop by during office hours or the lab session to talk over things in detail.

Part 1

Question 1(a) Demand equations and initial equilibrium

The demand equations can be found by combining the information about the household's preferred combinations of X and Y with its budget constraint. Since the household likes to have 3 units of Y for each unit of X, it will always choose Y and X as follows:

$$Y = 3 * X$$

Inserting this into its budget constraint in place of Y:

$$M = P_x * X + P_y * Y$$

$$M = P_x * X + P_y * (3 * X)$$

$$M = (P_x + 3 * P_y) * X$$

Solving for X and then Y:

$$X = \frac{M}{P_x + 3 * P_y} \text{ (demand for X)}$$

$$Y = 3 * X$$

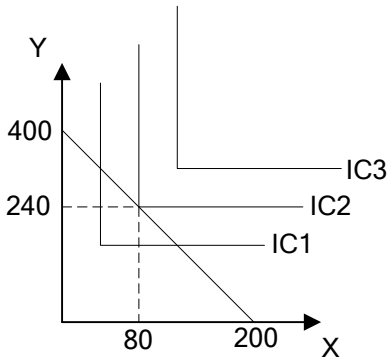
$$Y = \frac{3 * M}{P_x + 3 * P_y} \text{ (demand for Y)}$$

Calculating the initial consumption of each good:

$$X = \frac{\$40,000}{\$200 + 3 * \$100} = 80$$

$$Y = \frac{3 * \$40,000}{\$200 + 3 * \$100} = 240$$

Graphing the equilibrium:



Checking via the budget constraint:

$$M = \$200 \cdot 80 + \$100 \cdot 240 = \$16,000 + \$24,000 = \$40,000 \text{ (passes the check)}$$

Question 1(b) Effect of a reduction in the price of X

Calculating the new consumption of each good:

$$X = \frac{\$40,000}{\$100 + 3 \cdot \$100} = 100$$

$$Y = \frac{3 \cdot \$40,000}{\$100 + 3 \cdot \$100} = 300$$

Consumption of both goods rises: X increases by 20 and Y increases by 60.

Since the household has perfect complements preferences, the expenditure needed to return it to its original indifference curve can be found by computing the amount needed to purchase the original bundle (X=80, Y=240) at the new prices:

$$M3 = \$100 \cdot 80 + \$100 \cdot 240 = \$32,000$$

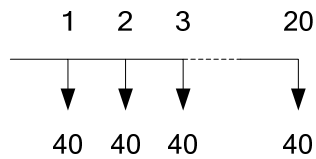
Computing the CV:

$$CV = M3 - M1 = \$32,000 - \$40,000 = -\$8,000$$

The CV is negative because the policy makes the household better off: returning it to the original IC would mean taking money away. Put another way, the policy makes households \$8,000 better off.

Question 1(c) Net present value of developing the technology

Cash flow for the development costs (in millions):



Computing the PV:

$$PV = \frac{40}{0.05} - \frac{(40/0.05)}{(1.05)^{20}}$$

$$PV = 800M - 301.5M = 498.5M$$

Since there are 10,000 households and each gains \$8000 once the technology has been developed, the total benefits in each year from 21 onward are $\$8,000 \times 10,000 = \$80M$. The cash flow (in millions) is thus:



Computing the PV in millions:

$$PV = \frac{(80/0.05)}{(1.05)^{20}} = \$603$$

The NPV, in millions, is:

$$NPV = \$603 - \$498.5 = \$104.5$$

Since the NPV is positive, it would be a good idea to proceed with the project: it generates more than enough benefits to cover its costs.

Part 2

The original equilibrium bundle will be:

$$A = 0.2 * 1000 / 4 = 50$$

$$B = 0.8 * 1000 / 8 = 100$$

The original utility will be needed for the CV calculation. Calculating it:

$$U = (50)^{0.2} * (100)^{0.8} = 87.055$$

After the taxes are imposed, the new equilibrium will be:

$$A = 0.2 * 1000 / 5 = 40$$

$$B = 0.8 * 1000 / 10 = 80$$

The revenue raised will be:

$$\text{Revenue} = \$1 * 40 + \$2 * 80 = \$200$$

The expenditure needed to reach the original IC at the new prices will be:

$$M3 = 87.055 * (\$5 / 0.2)^{0.2} (\$10 / 0.8)^{0.8} = \$1250$$

Computing the CV:

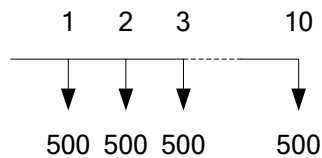
$$CV = M3 - M1 = \$1250 - \$1000 = \$250$$

The \$50 difference between the CV and the revenue is DWL.

Part 3

Question 3(a) Present value of alternative policies

The cash flow and PV for renovation, in millions:



$$PV = \frac{500}{0.05} - \frac{(500/0.05)}{(1.05)^{10}} = \$3,861\text{M or } \$3.861\text{B}$$

Moving to a new city would involve two costs: \$2.5B up front and \$100M a year forever. Computing the PV of the \$100M:

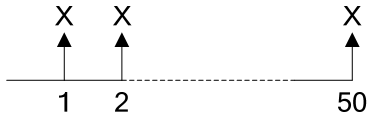
$$PV = \$100\text{M} / 0.05 = \$2\text{B}$$

The total PV of moving is thus \$2.5B + \$2B = \$4.5B.

Renovation is a better option: it saves \$4.5B - \$3.861B = \$639M.

Question 3(b) Tax credit needed to change the decision

The cash flow associated with the tax credit is shown below:



The PV will be:

$$PV = \frac{X}{0.05} - \frac{(X / 0.05)}{(1.05)^{50}}$$

Replacing $1/0.05$ with 20 and factoring out $20 * X$:

$$PV = 20 * X \left(1 - \frac{1}{(1.05)^{50}} \right)$$

Calculating the term in parentheses and simplifying again:

$$PV = 20 * X * 0.913 = 18.256 * X$$

To change the decision, the tax credit must be large enough to compensate for the \$639M cost advantage for renovation. That is:

$$PV \geq \$639M$$

$$18.256 * X \geq \$639M$$

$$X \geq \$35M$$

Thus, the tax credit would need to be at least \$35M per year to change the decision.