Peter J. Wilcoxen
Economics 410

The Maxwell School
Syracuse University

# Exam 2

Spring 2007

Version P Solution

**Question 1 (15 points)**

The government is considering whether or not to regulate the use of an industrial chemical. Nationwide, there are 100,000 workers in the industry that uses the chemical. A typical worker is exposed to a dose of 100 milligrams (0.1 gram). No epidemiological study has been done on the chemical but a clinical study is available. In the study, 400 mice were exposed to a dose that would be equivalent to 100 grams in a human, and there were 60 more cases of cancer than normal. The dose-response function is believed to be linear.

The government is considering two options: Policy A would cost $150 million and would ban the chemical completely; Policy B would cost $50 million and would restrict, but not ban, the chemical. Under Policy B, only 20,000 people would be exposed but those people would still receive the 0.1 gram dose.

Please calculate: (a) the number of expected cases of cancer in workers as a result of exposure to the chemical before regulation, (b) the net benefits of Policy A, and (c) the net benefits of Policy B. Then (d) explain which policy would be preferred and why. You may assume that the government uses $6 million as its estimate of the population's willingness to pay to save a statistical life.

(a) Risk in rats at 100 gram dose: 60/400 = 0.15 (or 15%).

Risk in humans at 0.1 gram dose: 0.15/1000 = 0.00015

Expected human cases: 0.00015*100,000 = 15

(b) Expected human cases under A: 0; lives saved under A: 15; benefits of A: 15*$6 million = $90 million; cost of A: $150 million; net benefits of A: -$60 million (90-150).

(c) Expected human cases under B: 0.00015*20,000 = 3; lives saved under B: 12; benefits of B: 12*$6 million = $72 million; net benefits of B: $22 million (72-50).

(d) Policy B is better: saves lives at a cost of $4.17 million when the population is willing to pay $6 million. Policy A is bad: costs $10 million per life saved but the population is only willing to pay $6 million.

**Question 2 (15 points)**

A state government would like to determine the value of a small park that is open only on weekends. No admission fee is currently charged and 490 people visit the park in a typical weekend. A researcher has interviewed a sample of the visitors and determined that they come from 5 geographic zones. The cost of a round trip to the park from each zone is shown in the table below, along with each zone's population and the number of people who visit the park from the zone.

| Zone | Travel Cost | Population | Visitors |
|---|---|---|---|
| A | $5 | 250 | 100 |
| B | $10 | 600 | 180 |
| C | $15 | 500 | 100 |
| D | $20 | 1100 | 110 |
| E | $25 | 2000 | 0 |

It is also known that the number of visits to the park (including people from all zones) is given by an equation of the form: P=A-B*Q, where P is the admission fee, Q is the number of visitors, and A and B are constants. The park is open 52 weekends a year and the government uses a 5% interest rate in PV calculations.

Please compute: (a) the number of people who would visit the park each weekend if a $5 admission fee were charged, (b) the amount of consumer surplus currently produced by the park each weekend, (c) amount of surplus produced over a year, and (d) the present value of keeping the land as a park forever. For simplicity, you can treat each year's annual surplus as arriving all at once: you do NOT need to worry about the fact that the weekends are scattered throughout the year.

(a) Computing the new number of visitors via a table:

| **Zone** | **Visitors** | **Population** | **Visit Rate** | **New Visit Rate** | **New Visitors** |
|---|---|---|---|---|---|
| A | 100 | 250 | 40% | 30% | 75 |
| B | 180 | 600 | 30% | 20% | 120 |
| C | 100 | 500 | 20% | 10% | 50 |
| D | 110 | 1100 | 10% | 0% | 0 |
| E | 0 | 1000 | 0% | 0% | 0 |
| **Total** | 490 | | | | 245 |

(b) Using the two data points to solve for A and B:

Actual data: 0 = A – B*490

Travel cost results: $5 = A – B*245

Using the first equation to eliminate A from the second:

A = B*490

$5 = B*490 – B*245 = B*245

B = 5/245 = 1/49 = 0.02041

A = (1/49)*490 = 10

CS = (1/2)*490*10 = $2,450 per weekend

(c) CS per year = 52*2,450 = $127,400

(d) CS forever = CS per year / r = $127,400/0.05 = $2.548 million

**Question 3 (15 points)**

A city is deciding what to do with a parcel of vacant land. The city is concerned about two periods, now (period 0) and a generation in the future (period 1). The interest rate between the two periods is 100%. The land is providing zero benefits now but the city believes that it might be valuable as a park in period 1. In particular, it believes that there is a 25% chance the land would produce $70 million of benefits as a park in period 1; otherwise, however, it would produce zero benefits. A developer would be willing to buy the land for $10 million in either period. Development of the land is irreversible.

Please: (a) calculate the net present value of leaving the park vacant in period 0, and (b) explain whether or not the city should sell the land to the developer in period 0 and why.

(a) If land is vacant in 1, the city can choose the highest value use at that time: the land can be a park if the high-value outcome occurs, or it could be sold to the developer for $10 million. Thus, the EV of vacant land in period 1 = 0.25*$70 million + 0.75*$10 million = $25 million

PV of EV1 = $25 million/(1+r) = $25 million/2 = $12.5 million

NPV relative to selling in 0: $12.5 million - $10 million = $2.5 million

(b) The city should not sell the land in period 0. Keeping the land vacant preserves the option to use it as a park in period 1. If it turns out that period 1 doesn't value the park highly, the land can be sold to the developer at that point.

**Question 4 (15 points)**

A river carrying 1000 units of water is used for three purposes: agriculture, drinking water, and recreation. The demand for agricultural water is given by W2Pa = 1000 – Qa, and the demand for drinking water is given by W2Pd = 4000 – 10*Qd. Both uses take water out of the river, and it does not return. Recreational benefits come from water left in the river. There are 200 recreational users of the river and each has a marginal benefit given by MBi = 10 – (1/20)*Qr, where Qr is the amount of water left in the river. Recreational use is non-rival.

Please calculate: (a), (b) and (c) the efficient quantities of water to allocate to the three uses. If the government wanted to allocate the water by charging agricultural and drinking water users for water, (d) what price should it charge? Recreational users would not have to pay.

(a), (b), (c) and (d) Since recreational benefits are non-rival, the marginal benefits, MBr, of providing Qr units of water in the river will be the sum of the marginal benefits received by each of the recreational users:

$$MBr = \sum_{i=1}^{200} MBi$$

Since the recreational users are identical, this simplifies to:

MBr = 200*( 10 – (1/20)*Qr )

MBr = 2000 - 10*Qr

For efficiency, the water should be allocated so that the marginal value of the last unit used in each application is equal:

MBr = W2Pa = W2Pd

In addition, the total use of water is constrained by the amount available:

Qr + Qa + Qd = 1000

There are multiple ways to solve for Qr, Qa and Qd. One approach is to treat them as three demand curves and solve for the market equilibrium:

MBr = 2000 – 10*Qr = P

W2Pa = 1000 – Qa = P

W2Pd = 4000 – 10*Qd = P

Qr = (2000 – P)/10

Qa = 1000 – P

Qd = (4000 – P)/10

QT = Qr + Qa + Qd

QT= (2000 – P)/10 + 1000 – P + (4000 – P)/10

QT = 1600 – P*( 0.1 + 1 + 0.1 )

QT = 1600 – 1.2*P

Inserting the actual amount of water available:

1000 = 1600 – 1.2*P

P = $500

The allocations that go with this:

Qr = (2000 – 500)/10 = 150

Qa = 1000 – 500 = 500

Qd = (4000 – 500)/10 = 350

Checking: 150 + 500 + 350 = 1000

An alternative way to solve the problem would be to set it up as a set of simultaneous equations:

MBr = W2Pa

W2Pa = W2Pd

Qr + Qa + Qd = 1000

A third approach would be to combine the two off-stream uses into a single demand curve for off-stream water:

Qa = 1000 – P

Qd = (4000 – P)/10

Qo = Qa + Qd = 1000 – P + (4000 – P)/10

Qo = 1400 – 1.1*P

This equation could now be used to balance off-stream and in-stream water:

P = (1400 – Qo)/1.1

2000 – 10*Qr = (1400 – Qo)/1.1

2200 – 11*Qr = 1400 – (1000 – Qr)

2200 – 11*Qr = 1400 – 1000 + Qr

1800 = 12*Qr

Qr = 150

Qo = 850

From here it is straightforward to calculate P, Qa and Qd.

**Question 5 (15 points)**

Consider the allocation of an exhaustible resource across three generations. Demand is the same in each of the periods but mining technology is improving and MEC is falling over time. The following information is available:

- Demand in each period: W2Pi = 1200 – Qi
- MEC in period 1: $300
- MEC in period 2: $200
- MEC in period 3: $100
- Resource available: 1600 units
- Interest rate: 100%

Please calculate: (a) the equilibrium royalty, extraction cost, price and quantity that would occur in each period, and summarize your results in a table. Then suppose that a backstop is available at a marginal cost of $700. Please calculate: (b) the new equilibrium royalty, extraction cost, price and quantity in each period, summarizing your results in a second table. Finally, calculate (c) the total amount of the resource produced via the backstop.

(a) Royalties in each period:

R1 = 1200 – Q1 – 300 = 900 – Q1

R2 = 1200 – Q2 – 200 = 1000 – Q2

R3 = 1200 – Q3 – 100 = 1100 – Q3

Solving for the Q's:

Q1 = 900 – R1

Q2 = 1000 – R2

Q3 = 1100 – R3

Summing:

QT = 900 – R1 + 1000 – R2 + 1100 – R3

QT = 3000 – R1 – R2 – R3

By arbitrage, the following must hold in equilibrium:

R2 = R1*(1+r) = 2*R1

R3 = R1*(1+r)^2 = 4*R1

Inserting into the QT equation:

QT = 3000 – R1 – 2*R1 – 4*R1 = 3000 – 7*R1

Using the information about known deposits:

1600 = 3000 – 7*R1

R1 = 200

Using this information, it is straightforward to solve for the remaining variables. The results are shown in the table below.

| **Period** | **R** | **MEC** | **P** | **Q** |
|---|---|---|---|---|
| **1** | 200 | 300 | 500 | 700 |
| **2** | 400 | 200 | 600 | 600 |
| **3** | 800 | 100 | 900 | 300 |

(b) With a backstop available at $700, the royalty in period 3 will drop to $700-$100 or $600. By arbitrage, the royalties in earlier periods will drop to $300 and $150. The other variables are straightforward to compute:

Pi = Ri + MECi

Qi = 1200 – Pi

| Period | R | MEC | P | Q |
|---|---|---|---|---|
| **1** | 150 | 300 | 450 | 750 |
| **2** | 300 | 200 | 500 | 700 |
| **3** | 600 | 100 | 700 | 500 |

(c) Total production in the backstop case is 1950. Of that, 1600 comes from the exhaustible resource and 350 comes from the backstop.

**Question 6 (15 points)**

Suppose that a supply of oil is to be allocated across two identical periods. In each period, the demand for oil is given by W2Pi = 1000 – (1/2)*Qi, and the marginal extraction cost is zero (MECi = 0). Initially, there are 2200 barrels available. However, it is possible to find additional barrels via exploration. The cost of drilling an exploratory well is $200. Seventy-nine percent (79%) of the time, no oil will be found, 20% of the time 1 barrel will be found, and 1% of the time 30 barrels will be found. The interest rate is 100%.

Please calculate: (a) the minimum oil price that will induce exploration; (b) the market equilibrium price and quantity in each period taking exploration into account, summarizing your results in a table; (c) the equilibrium amount of oil that will be found via exploration; and (d) and the expected number of wells that will be drilled.

(a) The expected value of drilling a well:

EV = 0.79*0 + 0.20*P + 0.01*30*P - $200

EV = (0.2 + 0.3)*P = 0.5*P - $200

The minimum price that would induce drilling is the value of P that makes the EV equal to 0:

EV = 0 = 0.5*P - $200

P = $400

(b) Since the MEC =0, the market equilibrium without exploration can be found as follows:

R1 = 1000 – (1/2)*Q1 – 0 = 1000 – (1/2)*Q1

R2 = 1000 – (1/2)*Q2 – 0 = 1000 – (1/2)*Q2

Q1 = 2000 – 2*R1

Q2 = 2000 – 2*R2

QT = Q1 + Q2 = 4000 – 2*(R1+R2)

R2 = R1*(1+r) = 2*R1

QT = Q1 + Q2 = 4000 – 6*R1

2200 = 4000 – 6*R1

R1 = 300

Calculating the other variables:

| Period | R | MEC | P | Q |
|---|---|---|---|---|
| **1** | 300 | 0 | 300 | 1400 |
| **2** | 600 | 0 | 600 | 800 |

With exploration, the price will be capped at $400. As a result, exploration will occur in period 2, driving down P2 = R2 = $400. Arbitrage will bring the price in period 1 down to $200. The equilibrium result will look like this:

| Period | R | MEC | P | Q |
|---|---|---|---|---|
| **1** | 200 | 0 | 200 | 1600 |
| **2** | 400 | 0 | 400 | 1200 |

(c) The amount of oil found by drilling will be total consumption, 2800, less the 2200 initially available, or 600 barrels.

(d) When N wells are drilled, the expected number of barrels found will be:

EV = 0.20*N + 0.01*30*N = 0.5*N

In order to find 600 barrels, therefore, N must satisfy the equation:

600 = 0.5*N

N = 1200