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Solution to Exam 2
Fall 2008

Here are notes on the solution. Some of the graphs may be omitted and the explanations are a bit terse. If you have any questions, please don't hesitate to stop by during office hours or the lab session to talk things over in detail.

Part 1: Perfect Complements

Question 1a

If the household wants exactly *a* units of X for each unit of Y, it will choose the quantities as follows:

$$X = a*Y$$

It faces the usual budget constraint:

$$M = Px*X + Py*Y$$

Substituting and solving for Y gives the demand equation:

$$M = Px*(a*Y) + Py*Y$$

$$M = (a*Px + Py)*Y$$

$$Y = \frac{M}{a*Px + Py}$$

Solving for the demand for X:

$$X = a*Y$$

$$X = \frac{a*M}{a*Px + Py}$$

To use these with the household data given in the exam, solve one of the demand equations for *a* in terms of the other variables. Either equation can be used but the algebra is easiest with Y:

$$Y = \frac{M}{a*Px + Py}$$

$$(a*Px + Py)*Y = M$$

$$a*Px*Y = M - Py*Y$$

$$a = \frac{M - Py * Y}{Px * Y}$$

If a household has perfect complements preferences, the value of *a* should be the same for both years of data. Checking household R:

R in 2006: a = (100-2*20)/(1*20) = 3

R in 2007: a = (112-2*14)/(2*14) = 3

That's enough to show that household R has perfect-complements preferences and a value of *a* equal to 3. The other two households do not have perfect-complements preferences: inserting their data gives different values for *a* in 2006 and 2007.

A quicker way to get to the same result is to check the ratios of X to Y for each of the households in each of the years. A household with perfect-complements preferences should have the same value of X/Y in each year. Checking:

Household	X/Y in 2006	X/Y in 2007	Perfect complements?
R	60/20 = 3	42/14 = 3	Yes
S	60/20 = 3	36/24 = 1.5	No
T	60/15 = 4	30/30 = 1	No

Question 1b

The 2008 equilibrium values of X and Y are:

$$X = \frac{a * M}{a * Px + Py} = \frac{3*140}{3*2+1} = 60$$

$$Y = \frac{M}{a * Px + Py} = \frac{140}{3*2+1} = 20$$

The graph is omitted but the x-intercept of the budget constraint would be 70 and its y-intercept would be 140. The indifference curves are right angles.

Question 1c

The new values of X and Y are:

$$X = \frac{a * M}{a * Px + Py} = \frac{3*140}{3*3+1} = 42$$

$$Y = \frac{M}{a * Px + Py} = \frac{140}{3*3+1} = 14$$

Since the household regards the goods as perfect complements, the expenditure needed to return the household to the original indifference curve is equal to the amount of money it would need to buy the original bundle (from 1b) at the new prices:

M3 = $3*60 + $1*20 = $200

Computing the CV:

CV = M3 – M1 = $200 - $140 = $60

As a percent of 2008 income:

$60/$140 = 43%

Part 2: Cobb-Douglas

Question 2a

Solving the demand equation for X for g:

g = Px*X/M

Calculating g for household S (skipping R since it has perfect-complements preferences) in both years:

S in 2006: g = $1*60/100 = 0.6

S in 2007: g = $2*36/120 = 0.6

Since the values are identical, S has Cobb-Douglas preferences. Doing the same calculation for household T would show that it does not have Cobb-Douglas preferences: the values would be different for the two years.

Deriving the expenditure function:

$$U = X^{g} Y^{1-g}$$

$$U = \left(\frac{g * M}{Px}\right)^{g} \left(\frac{(1-g) * M}{Py}\right)^{1-g}$$

$$U = \left(\frac{g}{Px}\right)^{g} \left(\frac{1-g}{Py}\right)^{1-g} M$$

$$M = U * \left(\frac{Px}{g}\right)^{g} \left(\frac{Py}{1-g}\right)^{1-g}$$

$$M = U * \left(\frac{Px}{0.6}\right)^{0.6} \left(\frac{Py}{0.4}\right)^{0.4}$$

Question 2b

The equilibrium values of X and Y can be calculated from the demand equations:

$$X = \frac{a * M}{Px} = \frac{0.6 * \$200}{\$2} = 60$$

$$Y = \frac{(1-a) * M}{Py} = \frac{0.4 * \$200}{\$1} = 80$$

The graph is omitted but the x-intercept of the budget constraint would be 100 and the y-intercept would be 200.

Question 2c

To find the value of Py that would cut Y from 80 to 40, invert the demand equation:

$$Y = \frac{0.4 * M}{Py}$$

$$Py = \frac{0.4 * M}{Y}$$

$$Py = \frac{0.4 * \$200}{40} = 2$$

Since Py would have to be $2, the tax would be $1. Since the quantity of Y would be 40 units, the tax would raise $1*40 = $40 of revenue.

To find the CV, the first step is to find the utility of the original consumption bundle (60 units of X and 80 units of Y):

$$U = X^{0.6} Y^{0.4} = 60^{0.6} * 80^{0.4} = 67.3173$$

Inserting this and the new prices (Px = $2 and Py = $2) into the expenditure function:

$$M3 = 67.3173 * \left(\frac{\$2}{0.6}\right)^{0.6} \left(\frac{\$2}{0.4}\right)^{0.4} = \$264$$

Computing the compensating variation:

CV = $264 - $200 = $64

The CV is substantially larger than the revenue raised ($64 vs. $40) and the difference is deadweight loss. Expressed as a percent of income the CV is $64/$200 = 32%.

Part 3: Other Preferences

Using the demand equation, the value of X after the tax would be:

$$X = \frac{M * Py}{Px * (Px + Py)} = \frac{\$50 * \$1}{\$2 * (\$2 + \$1)} = \frac{50}{6} = 8.33$$

Since the tax rate is $1, it will raise $1*8.33 = $8.33 of revenue. The value of Y (not required but handy for the last part of the problem) will be:

$$Y = \frac{M * Px}{Py * (Px + Py)} = \frac{\$50 * \$2}{\$1 * (\$2 + \$1)} = \frac{100}{3} = 33.33$$

To compute the compensating variation, the first step is to find the original bundle:

$$X = \frac{M * Py}{Px * (Px + Py)} = \frac{\$50 * \$1}{\$1 * (\$1 + \$1)} = \frac{50}{2} = 25$$

$$Y = \frac{M * Px}{Py * (Px + Py)} = \frac{\$50 * \$1}{\$1 * (\$1 + \$1)} = \frac{50}{2} = 25$$

Computing the initial utility:

$$U = \left(X^{0.5} + Y^{0.5}\right)^2 = \left(25^{0.5} + 25^{0.5}\right)^2 = (5 + 5)^2 = 100$$

Using the expenditure function to find M3, the compensated expenditure:

$$M3 = U * \left(\frac{Px * Py}{Px + Py}\right) = 100 * \left(\frac{\$2 * \$1}{\$2 + \$1}\right) = \$66.67$$

Computing the compensating variation: CV = M3 – M1 = $66.67 - $50 = $16.67. The CV is twice the revenue raised by the tax; the difference is deadweight loss.

The indifference curves are much flatter (more like perfect substitutes) than Cobb-Douglas. With Cobb-Douglas preferences, the household would keep its budget shares constant and would still spend half of its money on X after the price increase. Since the price doubles, a Cobb-Douglas household would cut its consumption of X by 50%. However, X falls considerably more than that: it is down by two-thirds: from 25 down to 8.33. Also, a Cobb-Douglas household would not change its consumption of Y since Py hasn't changed. In this case, however, consumption of Y increases sharply. The results for X and Y both suggest that the household's preferences are closer to perfect substitutes than perfect complements.