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PPA 990 Natural Resource Economics

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Exercise 2

Consider the problem of managing a fishery consisting of two species: plankton and humpback whales. Plankton is consumed by whales and whales are consumed by humans. Suppose that in the absence of whales the stock of plankton changes according to the function:

$$G(p) = gp(K - p)$$

where p is the stock of plankton and g and K are constants determined by plankton biology. In addition, suppose that w whales consume an amount of plankton, x, given by the expression:

$$x = apw$$

where a is a constant. Finally, suppose that in the absence of whaling changes in the whale stock depend on the number of whales and the amount of plankton they consume:

$$H(x, w) = \gamma(x - hw)$$

where h and γ are constants determined by the biology of whales.

(a) Find the equations of motion for the populations of whales and plankton in the absence of human intervention.

(b) Find an expression for the steady state population of each species in terms of biological constants. Discuss how changes in those constants would affect the steady state populations.

(c) Construct a phase diagram illustrating the dynamics of the two populations. Put the stock of plankton on the horizontal axis and the stock of whales on the vertical axis. Show the steady state, the two isoclines, the directions of motion in each quadrant, and a representative path from an initial point with few whales to the steady state.

(d) Finally, consider what happens when whaling is introduced. If the number of whales caught by humans is y, the derivative of the stock of whales becomes:

$$\frac{dw}{dt} = H(x, w) - y$$

Suppose the long run objective of whalers is to catch the maximum sustainable yield of whales. Find the optimal stocks of whales and plankton and compare these to their pre-whaling counterparts.