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PPA 990 Natural Resource Economics

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Exercise 3

Consider an ecosystem consisting of trees and spotted owls. In the absence of human intervention, the populations of trees (T) and owls (W) evolve according to the equations:

$$\frac{dT}{dt} = 10T - \frac{T^2}{100}$$

$$\frac{dW}{dt} = \frac{WT}{1000} - \frac{W^2}{500} - 45$$

(a) Solve for the two steady-state populations of owls and trees. Explain intuitively why there are two and discuss the key difference between them.

(b) Draw an appropriate phase diagram for the regions near the steady states. Put the stock of trees on the horizontal axis. Show the steady states, the isoclines and the directions of motion. Keep in mind that the two steady states differ in an important way: the phase diagram in the regions near each one will differ as a result. Because there are two steady states, you may find it helpful to plot the actual equations for the isoclines rather than just using partial derivatives to deduce their slopes.

(c) Now imagine that the owner of the forest begins harvesting trees at a rate x which is constant over time. Suppose the owner sets x to the maximum sustainable yield of trees. Find the new steady state and compare it to the old one. Discuss.

(d) Suppose the owl population is subject to normally distributed random shocks with mean zero and standard deviation equal to 25, and that this forest is the owl's only habitat. If you were in charge of protecting the owl from extinction with reasonable probability, would you restrict timber harvests, and if so, to what level? In answering this, you may interpret “reasonable probability” to mean that there is less than a 2.5% chance or so that the owls will fall below the minimum viable population in any single year.