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PPA 990 Natural Resource Economics

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Exercise 6

A price-taking firm operates a mine and produces ore at rate $x(t)$. The firm's revenue on this output is $px(t)$, where p is the price of ore. In addition, suppose the firm's costs are given by cx^2, where c is a constant. Finally, at any point in time let the ore remaining in the mine be $s(t)$. The evolution of s is governed by the following equation: $ds/dt = -x(t)$.

Set up and solve the firm's optimization problem assuming its objective is to maximize the present value of the sum of: (1) profits on the mine over the period from t=0 to 20, and (2) selling all remaining ore in year 20 for v dollars per unit of the stock. Note that the firm sells the ore without extracting it from the ground: it receives $v*s$ in year 20 and does *not* have to pay the extraction costs. Take the interest rate to be 5%, the value of p to be \$16, the value of v to be \$10, the value of c to be \$1, and the initial stock of ore to be 120 units.

(a) Set up the optimization problem, construct the Hamiltonian function, and then find the firm's first order conditions.

(b) Use the first order conditions to derive explicit closed-form equations for $x(t)$ and $s(t)$.

(c) Calculate the values of $x(0)$, $x(20)$ and $s(20)$ and then plot the integral curves for x and s.