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Exam 2 Fall 2012

DO NOT OPEN THIS EXAM UNTIL YOU ARE TOLD TO DO SO.

Instructions

- 1. Write your SUID in the upper right corner of this exam. Do NOT write your name.
- 2. SHOW ALL YOUR WORK. Answers without supporting work will receive little or no credit.
- 3. Do all your work on this exam. If you need extra space, write on the backs of the pages. However, if you do write an answer on the back of a page, *be sure you've noted that near the question*.
- 4. There are 72 points on the exam and you'll have 80 minutes to work on it. Budget your time accordingly.
- 5. Most of the questions have several steps. For clarity, they are indicated by <u>1a</u>, <u>1b</u>, etc. in the text.
- 6. Some algebraic relationships for exponents:

$$(AB)^{c} = A^{c}B^{c}, \quad A^{c}A^{d} = A^{c+d}, \quad (A^{c})^{d} = A^{cd}$$

7. The general form of the Cobb-Douglas utility function and its demand equations:

$$U = X^{a}Y^{1-a} \qquad X = \frac{aM}{P_{X}} \qquad Y = \frac{(1-a)M}{P_{Y}}$$

Question 1 (12 points)

One of the households in the table below has Cobb-Douglas preferences. Ia Using the data for 2010 and 2011, please determine which one and calculate the value of a for that household. Next, Ib starting from the data for 2011, suppose the government imposes a new policy that places a \$3 tax on good X but also provides the household with \$300 of extra income. You may assume the supply of good X is perfectly elastic so that its price rises to \$15. Determine the household's new consumption of X and Y. Ic Draw the household's new budget constraint and include the numerical values of its intercepts. Also sketch several of its indifference curves and show its equilibrium on the diagram. Be sure to show your work and label everything.

House	Year	Income	Px	Py	Qx	Qy
А	2010	1200	10	15	72	32
	2011	1440	12	12	60	60
В	2010	1000	10	15	40	40
	2011	900	12	12	30	45
С	2010	1215	10	15	81	27
	2011	1344	12	12	84	28

Question 2 (12 points)

2a Please derive the expenditure function for a household with Cobb-Douglas preferences. Be sure to show all the steps, not just the final result. Now **2b** compute the expenditure needed under the new policy for the household to be exactly as well off as it was in 2011 before the policy was implemented. **2c** How much better or worse off is the household than it was in 2011 before the policy was adopted? Be sure to be clear about whether the household has gained or lost.

Question 3 (12 points)

A household regards X and Y as perfect complements and always buys b units of good X for each unit of good Y. 3a Please derive the household's demand equations for X and Y in terms of b, Px, Py and income M. Be sure to show the steps involved, don't just write down the equations. Now 3b determine which one of the households in the table below (same as the previous table) has perfect complements preferences and calculate the value of b. Finally, 3csuppose a tax policy raises the price of X to \$15 and provides the household with an extra \$309 of income (same tax as in Question 1 but a different income boost). Determine the household's new consumption of X and Y and calculate the policy's compensating variation.

House	Year	Income	Px	Py	Qx	Qy
А	2010	1200	10	15	72	32
	2011	1440	12	12	60	60
В	2010	1000	10	15	40	40
	2011	900	12	12	30	45
С	2010	1215	10	15	81	27
	2011	1344	12	12	84	28

Question 4 (12 points)

A household buys two goods, X and Y, and its preferences can be represented by the utility function shown below. (This is an example of a Stone Geary utility function, which is essentially an improved version of Cobb Douglas). Also shown are the household's demand equations and its expenditure function:

$$U = (X - 100)^{0.5} (Y + 50)^{0.5} \qquad X = 50 + \frac{0.5 * M + 25P_y}{P_x}$$
$$M = 100P_x - 50P_y + 2 * U * P_x^{0.5} P_y^{0.5} \qquad Y = -25 + \frac{0.5 * M - 50P_x}{P_y}$$

Initially, Px=\$16, Py=\$16 and M=\$3200. The government is considering a policy that would (a) place a \$9 tax on good X that would raise its price to \$25 and (b) give the household an extra \$100 of income. The supply of X is perfectly elastic.

[4a] Please calculate the initial equilibrium before the policy is enacted. **[4b]** What is the household's new equilibrium? **[4c]** Please calculate the policy's compensating variation. Is the household better or worse off? **[4d]** How much revenue does the tax raise?

Question 5 (12 points)

5a Sketch an appropriate diagram to illustrate your results from Question 4. Then 5b calculate the income and substitution effects associated with the policy and show them in the diagram.

Question 6 (12 points)

Finally, suppose that there are actually two households in the economy: L and H. The households have identical preferences that can be represented by the Stone Geary equations in Question 4. However, the households have different initial income: household L is identical to the household above and has \$3200 before the policy. Household H has much more income: \$6400 before the policy. The policy is the same as before: the price of X rises by \$9 to \$25, and each household receives a \$100 supplement to its income.

6a Compute the new equilibrium for household H. Using your new results for H and your previous results for L please indicate **6b** the share of each household's budget spent on good X when the policy is in place and **6c** the amount of revenue raised by the tax from each household. **6d** Briefly discuss whether or not the tax is regressive. Be sure to be quantitative. **6e** What characteristic of the demand for X causes that result? (In case you're curious, this is the big difference between the Stone Geary and Cobb Douglas utility functions: this phenomenon cannot happen with Cobb Douglas)