

Monopoly with Diminishing Returns to Scale

Notes on Solution

Production function: $Q = K^{0.25} L^{0.25}$

Pk 100
Pl 16

Question 1

$$Q = K^{0.25} L^{0.25}$$

$$Q/K^{0.25} = L^{0.25}$$

$$(Q/K^{0.25})^{(1/0.25)} = L$$

Case for Q = 10

Q	K	L	TC	AC
10	35	285.71	8,071	807.1
10	36	277.78	8,044	804.4
10	37	270.27	8,024	802.4
10	38	263.16	8,011	801.1
10	39	256.41	8,003	800.3
10	40	250.00	8,000	800.0
10	41	243.90	8,002	800.2
10	42	238.10	8,010	801.0
10	43	232.56	8,021	802.1
10	44	227.27	8,036	803.6
10	45	222.22	8,056	805.6

Case for Q = 15

Q	K	L	TC	AC
15	85	595.59	18,029	1,202.0
15	86	588.66	18,019	1,201.2
15	87	581.90	18,010	1,200.7
15	88	575.28	18,005	1,200.3
15	89	568.82	18,001	1,200.1
15	90	562.50	18,000	1,200.0
15	91	556.32	18,001	1,200.1
15	92	550.27	18,004	1,200.3
15	93	544.35	18,010	1,200.6
15	94	538.56	18,017	1,201.1
15	95	532.89	18,026	1,201.8

Case for Q = 20

Q	K	L	TC	AC
20	155	1,032.3	32,016	1,600.8
20	156	1,025.6	32,010	1,600.5
20	157	1,019.1	32,006	1,600.3
20	158	1,012.7	32,003	1,600.1
20	159	1,006.3	32,001	1,600.0
20	160	1,000.0	32,000	1,600.0
20	161	993.8	32,001	1,600.0
20	162	987.7	32,002	1,600.1
20	163	981.6	32,006	1,600.3
20	164	975.6	32,010	1,600.5
20	165	969.7	32,015	1,600.8

Question 2

Checking against the given TC function:

$$TC = 80 \cdot Q^2$$

Q	TC
10	8000
15	18000
20	32000

Questions 3 and 4

Demand: $P = A - B \cdot Q$

A	4000
B	50

Q	P	TR	TC	AR	AC	AR-AC	MR	MC	MR-MC	Profit
10	3500	35000	8000	3500	800	2700				27000
11	3450	37950	9680	3450	880	2570	2950	1680	1270	28270
12	3400	40800	11520	3400	960	2440	2850	1840	1010	29280
13	3350	43550	13520	3350	1040	2310	2750	2000	750	30030
14	3300	46200	15680	3300	1120	2180	2650	2160	490	30520
15	3250	48750	18000	3250	1200	2050	2550	2320	230	30750
16	3200	51200	20480	3200	1280	1920	2450	2480	-30	30720
17	3150	53550	23120	3150	1360	1790	2350	2640	-290	30430
18	3100	55800	25920	3100	1440	1660	2250	2800	-550	29880
19	3050	57950	28880	3050	1520	1530	2150	2960	-810	29070
20	3000	60000	32000	3000	1600	1400	2050	3120	-1070	28000
21	2950	61950	35280	2950	1680	1270	1950	3280	-1330	26670
22	2900	63800	38720	2900	1760	1140	1850	3440	-1590	25080
23	2850	65550	42320	2850	1840	1010	1750	3600	-1850	23230
24	2800	67200	46080	2800	1920	880	1650	3760	-2110	21120
25	2750	68750	50000	2750	2000	750	1550	3920	-2370	18750
26	2700	70200	54080	2700	2080	620	1450	4080	-2630	16120
27	2650	71550	58320	2650	2160	490	1350	4240	-2890	13230
28	2600	72800	62720	2600	2240	360	1250	4400	-3150	10080
29	2550	73950	67280	2550	2320	230	1150	4560	-3410	6670
30	2500	75000	72000	2500	2400	100	1050	4720	-3670	3000
31	2450	75950	76880	2450	2480	-30	950	4880	-3930	-930
32	2400	76800	81920	2400	2560	-160	850	5040	-4190	-5120
33	2350	77550	87120	2350	2640	-290	750	5200	-4450	-9570
34	2300	78200	92480	2300	2720	-420	650	5360	-4710	-14280

To maximize output without running a loss, produce 30 jumps and charge \$2500 each. Profit will be \$3000. Could also produce 30 jumps and charge only \$2400; would eliminate the profit.

To maximize profit, produce 15 jumps and charge \$3250 for each one. Profit would be \$30,750.

Question 5

If Andrea acted like a competitive market and produced where $P = MC$ (approximately) she would choose Q where $P - MC$ is as small as possible. That can be found in the table below, which reproduces some of the numbers from above:

Q	P	TR	TC	MC	P-MC	Profit
10	3500	35000	8000			27000
11	3450	37950	9680	1680	1770	28270
12	3400	40800	11520	1840	1560	29280
13	3350	43550	13520	2000	1350	30030
14	3300	46200	15680	2160	1140	30520
15	3250	48750	18000	2320	930	30750
16	3200	51200	20480	2480	720	30720
17	3150	53550	23120	2640	510	30430
18	3100	55800	25920	2800	300	29880
19	3050	57950	28880	2960	90	29070
20	3000	60000	32000	3120	-120	28000
21	2950	61950	35280	3280	-330	26670
22	2900	63800	38720	3440	-540	25080
23	2850	65550	42320	3600	-750	23230
24	2800	67200	46080	3760	-960	21120
25	2750	68750	50000	3920	-1170	18750

She would produce 19 jumps and charge between \$2960 and \$3050 for them. This would be more efficient because $W2P$ for the last unit is equal (approximately) to MC . In the two monopoly cases she either stops producing too soon (profit maximization) or goes on producing too far (output maximization).

Can see this in the following table, which shows the surplus that could be gained for each unit produced beyond 15. The surplus is the difference between $W2P$ and MC . Units up to 19 create gains but after that additional units are net losses.

Q	W2P	MC	W2P-MC
16	3200	2480	720
17	3150	2640	510
18	3100	2800	300
19	3050	2960	90
20	3000	3120	-120
21	2950	3280	-330
22	2900	3440	-540
23	2850	3600	-750
24	2800	3760	-960
25	2750	3920	-1170