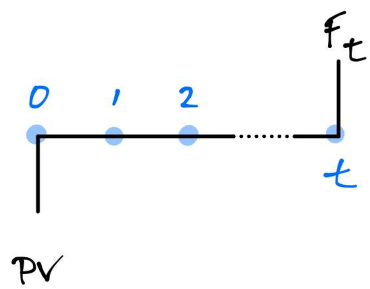
E: Present value (PV) refresher 1

Fundamental interpretation of PV:

- The amount of **money needed in a bank account today** (time 0) in order to **deliver a target sequence of payments** in the future.

Formula 1: Single payment:



$$PV = \frac{F_t}{(1+r)^t}$$

Example 1: PV of a single payment:

F	$10k
t	Year 5
r	10%

PV = $10,000/(1.1)^5 = $6,209

Example 2: using PV as a benchmark for evaluating policies:

Proposed policy:

Cost	$3000 today
Delivers	$5000 in year 4

Cost of alternative using a bank account at r=10%?

PV = $5000/(1.1)^4 = $3415

Conclusion:

Project is $415 cheaper than the bank.

Expressing via net present value (NPV):

NPV = PV of benefits - PV of costs

NPV = $3415 - $3000 = $415

Project produces a net gain of $415

Example 3: arbitrage trading:

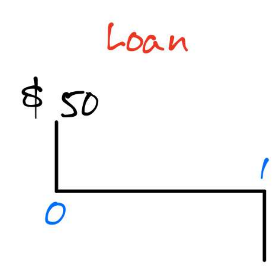
Suppose know price of oil is rising:

$$P_0 = \$50$$
$$P_1 = \$60$$
$$r = 10\%$$

Arbitrage trade:

At t=0:

- Borrow $50

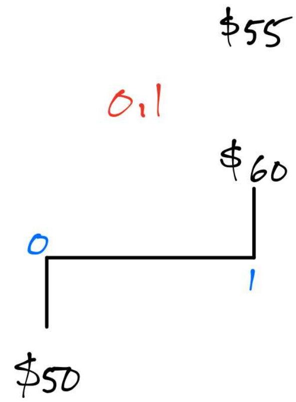


At t=1:

- Repay loan

and **and**

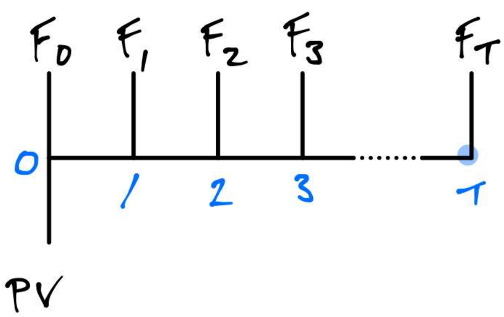
- Buy oil
- Sell oil



NPV = \$60/1.1 - \$55/1.1 = \$4.55

Profitable: returns exceed interest cost

Formula 2: extension to streams with multiple payments from 0 to T:

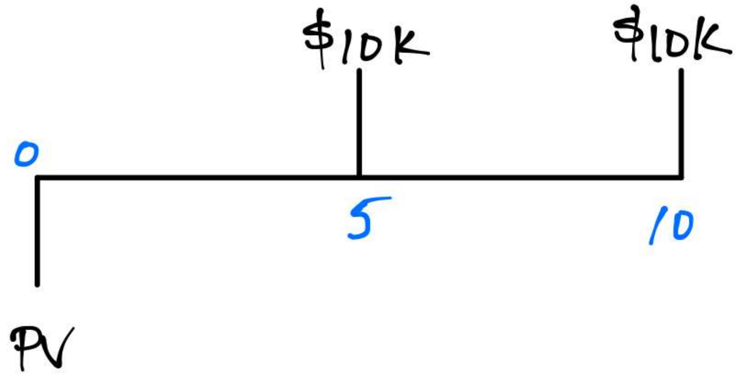


$$PV = \sum_{t=0}^{T} \frac{F_t}{(1+r)^t}$$

PV of the stream is the sum of the individual PVs

Example 4: two payments

Payments each \$10,000
One in year 5, one in year 10
r = 5%



PV = $10,000/1.05^5 + $10,000/1.05^10 = $13,974