E: Imperfect information and health policy

Imperfect tests are everywhere in medicine and health

Both kinds of errors:

False negatives: Miss disease when present

False positives: Report disease when absent

Three kinds of costs:

- Cost of test itself
- Costs of false negatives:
  Fail to treat disease or treat it late
- Costs of false positives:
  Anxiety
  Extra tests (e.g., biopsies)
  Unnecessary treatments (e.g., surgery)

  Can be a significant problem if **underlying risk is low**
  - Often the case with **screening** tests
  - Examples: prostate-specific antigen (PSA), mammograms

Example test:

Condition:

C: cancer

N: no cancer

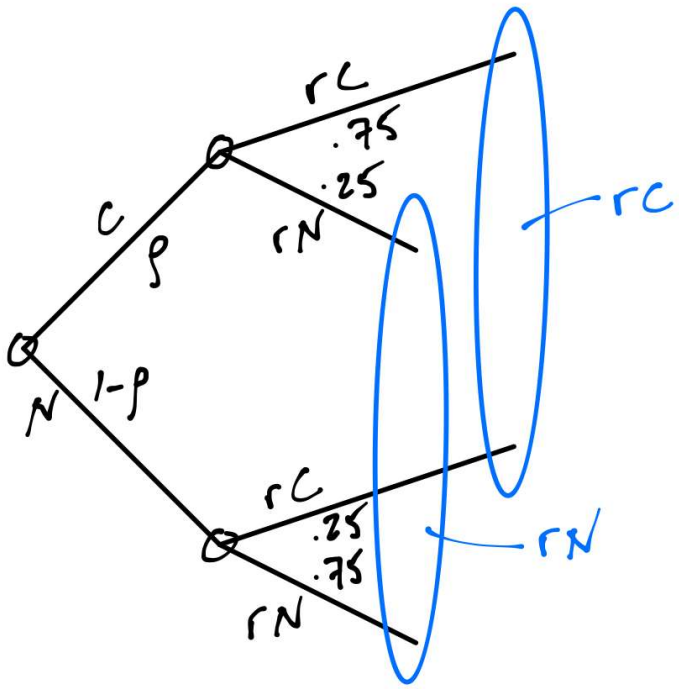
Test performance:

25% chance of false **positive**: **rC** when true condition is **N**

25% chance of false **negative**: **rN** when true condition is **C**

Question: treat if rC?

Let probability of C be ρ



Unconditional probabilities:

State	Report	Prob
C	rC	$\rho * 0.75$
C	rN	$\rho * 0.25$
N	rC	$(1-\rho) * 0.25$
N	rN	$(1-\rho) * 0.75$

Probability of reports:

Report	Probability	Simplifying
rC	$\rho * 0.75 + (1 - \rho) * 0.25$	$0.25 + 0.5\rho$
rN	$\rho * 0.25 + (1 - \rho) * 0.75$	$0.75 - 0.5\rho$

Conditional probability of C given rC:

$$\Pr(C|rC) = \frac{\rho * \ 0.75}{0.25 + 0.5\rho}$$

Impact of ρ:

Case 1: C is **certain** (test is redundant)

$$\rho = 1$$
$$\Pr(C|rC) = \frac{1 * 0.75}{0.25 + 0.5} = 1$$

Decision: treat

Case 2: C is **common** (half of population has it)

$$\rho = 0.5$$
$$\Pr(C|rC) = \frac{0.5 * 0.75}{0.25 + 0.5 * 0.5} = 0.75$$

Decision: Probably treat (depends on costs)
Overtreat 25% of patients

Case 3: C is **unusual** (one out of ten has it)

$\rho = 0.1$

$$\Pr(C|rC) = \frac{0.1 * 0.75}{0.25 + 0.5 * 0.1} = 0.25$$

Decision: Dicey to treat (depends on costs)
Would overtreat 75% of patients

Case 4: C is **rare** (one out of a thousand has it)

$\rho = 0.001$

$$Pr(C|rC) = \frac{0.001 * 0.75}{0.25 + 0.5 * 0.001} \approx 3\rho = 0.003$$

Decision: ??

How high are the stakes if we treat when rC?

Number of errors of each type per 100,000 tests:

State	Report	Unconditional probability	Per 100,000
C	rC	0.75*0.001 = 0.00075	75
C	rN	0.25*0.001 = 0.00025	25
N	rC	0.25*(1-0.001) = 0.24975	24,975
N	rN	0.75*(1-0.001) = 0.74925	74,925

Almost **25,000** people **treated unnecessarily**
… **and** we still **miss 25 people with C**

Important issue with many screening tests:

- Most patients are healthy

- Need **false positive rate to be very low**

Exercise on GC