E: Risk aversion, part 1

Modeling choice when individuals care about **variability** of payoffs

So far used EV to evaluate uncertain events:

- Identifies efficient outcomes
- Desirable goal in many circumstances

However, individuals may care about variability, not just EV

Example:

- Person has $100
- Offered two options:

A: Get $10 for certain

B: Coin toss:
Heads get $120
Tails lose $100

EV of A:
1*(10) = 10

EV of B:
0.5*(120) + 0.5*(-100) = 60 - 50 = 10

If prefers A to B:
Displays *risk aversion*

Risk aversion:

A decision maker displays risk aversion if:

Among bundles with same EV, prefers options with less variability

Example above:

Option	EV	σ
A:	\$10	\$0
B:	\$10	\$110

Common approach for modeling is *expected* ***utility***:

Slight extension of EV

Expected utility model:

Also known as Von Neumann-Morgenstern preferences

Key idea:

Ex ante utility of a gamble is its ***expected*** *ex post* utility

Formally:

N	Number of possible states
c_i	Gross consumption in state i
$u(c_i)$	Ex post utility of having c_i
ρ_i	Probability of state i

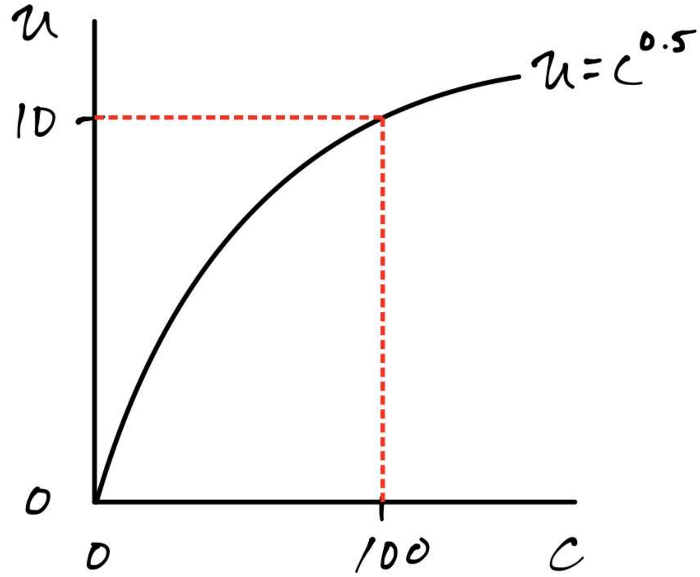
Expected utility, EU:

$$EU = \sum_{i=1}^{N} \rho_i \cdot u(c_i)$$

Applying to example:

Suppose $u(c_i) = (c_i)^{0.5}$

Consumption subject to diminishing returns:



Initially, person has $100

Option A: $10, sure thing

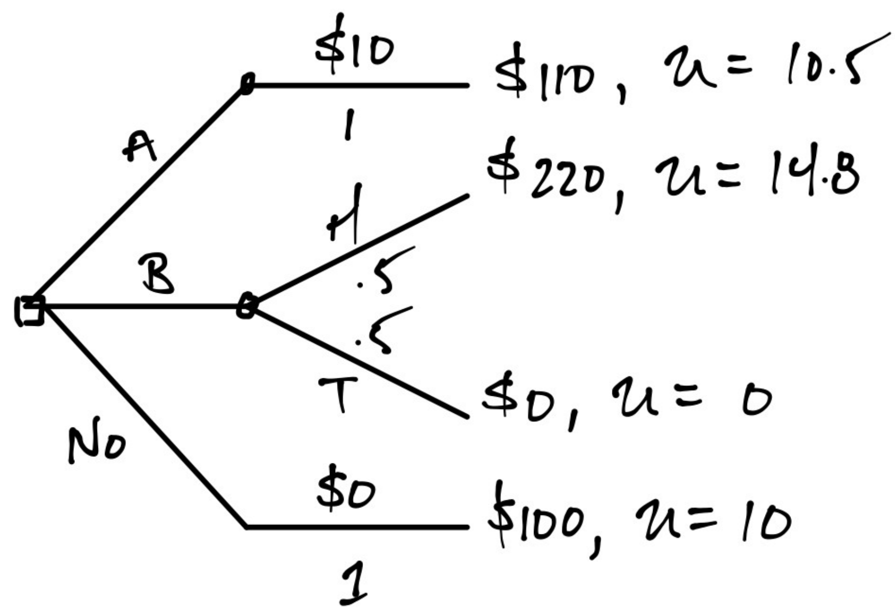
$$c = \$100 + \$10 = \$110$$

$$u(110) = (110)^{0.5} = 10.5$$

Option B: coin flip

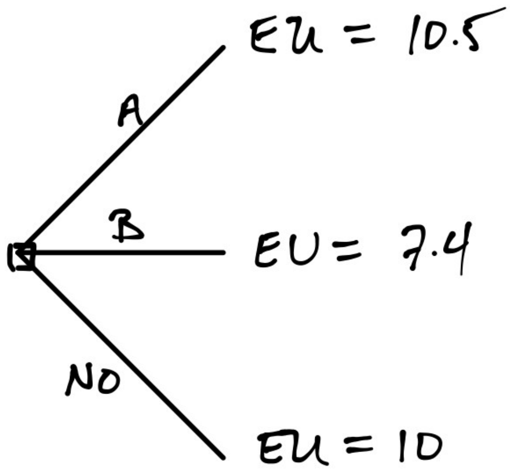
State	c_i	$u = (c_i)^{0.5}$
H	\$100 + \$120 = \$220	$u(220) = 14.8$
T	\$100 - \$100 = \$0	$u(0) = 0$

Decision tree with ex post payoffs:



Working backward to calculate ex ante expected utilities:

$$EU_B = 0.5 * (14.8) + 0.5 * (0) = 7.4$$

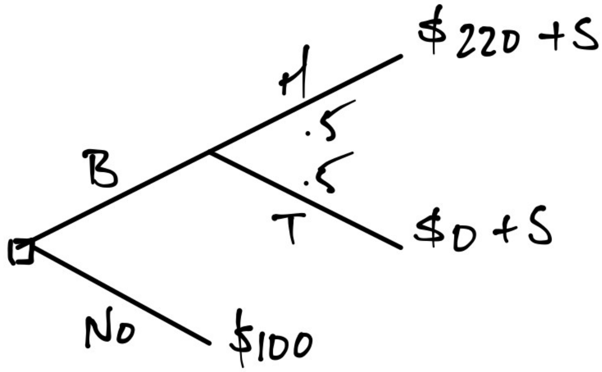


Choice: option A

Would have to **pay** agent to take B rather than not betting:

How much?

Suppose subsidy is S



$$EU_B = 0.5 * (220 + S)^{0.5} + 0.5 * (S)^{0.5}$$

$$EU_N = 1 * (100)^{0.5} = 10$$

Indifferent when $EU_B = EU_N$:

$$0.5 * (220 + S)^{0.5} + 0.5 * (S)^{0.5} = 10$$

$$S = 20.25$$

Compensation needed to accept risk = $20.25
=> one measure of the "risk premium"

Analogous to compensating variation (CV)

Will refer to as a *participation constraint*:
Agent must be at least as well off as not participating

Can look at a second way:

What is the **certain** payoff **equivalent** to the gamble?
Known as the certainty equivalent: CE

$$EU_B = 0.5 * (220)^{0.5} + 0.5 * (0)^{0.5}$$

$$EU_{CE} = 1 * (CE)^{0.5}$$

Find CE such that: $EU_B = EU_{CE}$

$$0.5 * (220)^{0.5} + 0.5 * (0)^{0.5} = 1 * (CE)^{0.5}$$

$$7.416 = CE^{0.5}$$

$$CE = 7.416^2 = 55$$

If forced to gamble (unavoidable risk):

- Give up $100
- End up as happy as having $55
- Like losing $45

Example application: renters insurance

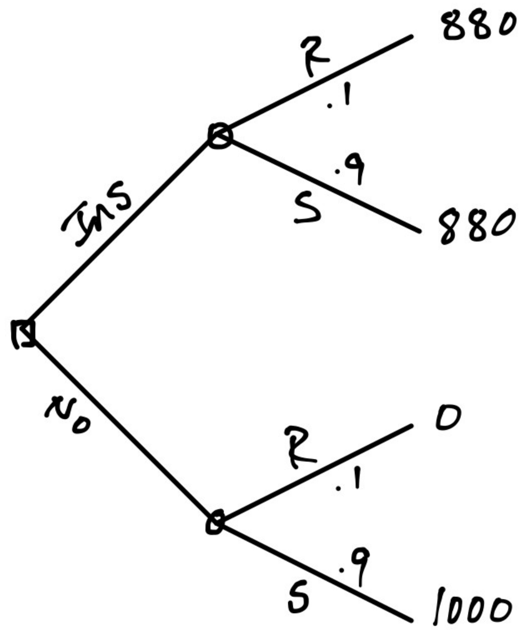
Initial cash:	$1000
States:	R = robbed, S = safe
Risk of R:	10%
Insurance premium:	$120
Ex post utility:	$u = c_i^{0.5}$

Actuarially fair policy for reference:

Expected claim: 0.1*1000 + 0.9*0 = 100

Fair premium: $100

Renter's decision tree:



1000-1000+1000-120 = 880
U = 29.7

1000-120 = 880
U = 29.7

1000-1000 = 0
U = 0

1000
U = 31.6

Evaluating:

$$EU_{INS} = 0.1 * (29.7) + 0.9 * (29.7) = 29.7$$

$$EU_{No} = 0.1 * (0) + 0.9 * (31.6) = 28.5$$

Would insure even though policy is not actuarially fair

Exercise on GC