Math Refresher 2022
Session 4: Simultaneous Equations

Application: Uncertain Subsidy

The equations below come from a model of how investors might react to a subsidy S that could be repealed with probability ρ:

$$S = \$50 + \$500\rho$$

$$\rho = \frac{S}{\$1500}$$

(a) Solve for the values of S and ρ.

(b) Sketch a graph of the two equations. Put S on the vertical axis and ρ on the horizontal axis.

The details are beyond the scope of today's session, but what your results show is the minimum subsidy S that will induce investors to make the desired investment given that they believe the probability that the policy will be repealed is given by the second equation. Subsidies below that value will have no effect.

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Application: Consumer Choice

The equations below are an example of a particular kind of consumer behavior when an individual is choosing a mix of goods x and y. The first equation relates to the person's preferences and the second represents their budget constraint when the price of x is \$4, the price of y is \$10, and the person is spending $\$100$.

(a) Solve for the values of x and y:

$$\frac{x}{y} = \frac{5}{2}$$

$$\$100 = \$4x + \$10y$$

(b) Sketch a graph of the two equations, label the intersection, and show the intercepts.

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Application: Intertemporal Consumer Choice

The equations below are an example of intertemporal choice when an individual is choosing a mix of consumption, c_0 and c_1, at two points in time, 0 and 1. The first equation relates to the person's preferences and the second represents their budget constraint. Solve for the values of c_0 and c_1:

$$\frac{c_0}{c_1} = \frac{1}{4}$$

$$c_0 + \frac{c_1}{2} = 150{,}000$$

(c) Sketch a graph of the two equations, label the intersection, and show the intercepts. Put c_0 on the horizontal axis and c_1 on the vertical axis.

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Application: Labor Market

The equations below represent a labor market with two employees and one employer. The first two show how the amount of hours supplied by each employee, q_1 and q_2, relates to the wage, w; the third equation shows how the employer's total demand for hours, q_m, relates on the wage; and the last shows how total hours are related to the hours supplied by the individual employees:

$$w = \frac{q_1}{20}$$

$$w = \frac{q_2}{32}$$

$$w = 76 - \left(\frac{1}{100}\right) q_m$$

$$q_m = q_1 + q_2$$

(a) Solve for w and the three q's. A good approach is to use the first two equations to eliminate q_1 and q_2 from the last equation, and then use that equation to eliminate q_m from the third equation to determine w.

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Application: Pollution Tax

The market for a particular good can be represented by the three equations below, where p^d is the price paid by buyers, t is a tax paid to the government, p^s is the price received by the sellers after the tax is deducted from p^d, and q is the quantity:

$$p^d = 300 - q$$

$$p^s = 2q$$

$$p^d - t = p^s$$

(a) Draw a graph showing the first two equations. Put p^d and p^s on the vertical axis (dollars) and q on the horizontal axis. Then set the tax t to 0 and solve for the quantity where the curves intersect, find the values of p^d and p^S, and then label the point accordingly.

(b) Now go back to the original 3 equations (before t was set to 0). Use the first two equations to eliminate p^d and p^S from the third equation and then solve for q as a function of tax t. The resulting equation shows how the equilibrium quantity changes with the tax.

(c) Sketch a graph of the equation from (b) showing how q depends on t. Put q on the vertical axis and t on the horizontal axis, and calculate the intercepts.

(d) Now suppose the government wants to impose a tax that will decrease q by 20% of its value from part (a). What should the tax be?

This kind of approach is used for calculating taxes on pollutants like carbon dioxide.