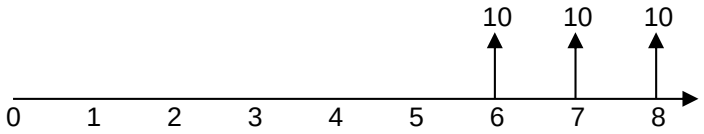
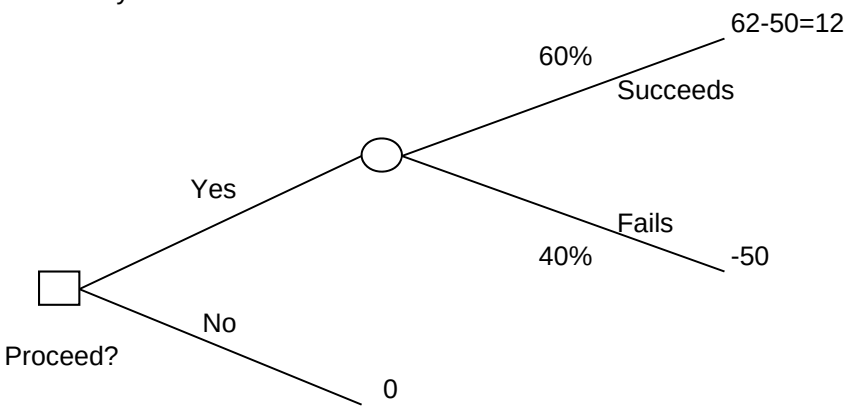
Fuel cell research under uncertainty
Notes on Solution

1 Cash flow diagrams and decision trees

Cash flow if the project succeeds



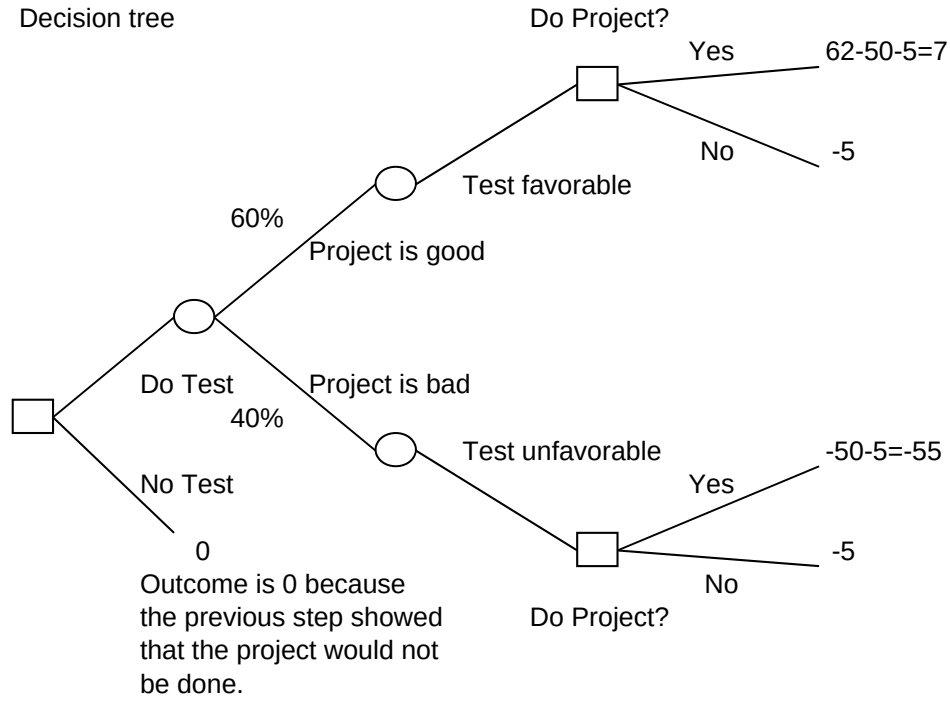
PV in year 5 = 100
PV in year 0 = 62



EV of project: 0.6*12 + 0.4*(-50) = -12.8 million

Don't do the research project because its expected value is negative. The EV is negative partly because the project's chance of success is not all that high (60%) and partly because the payoff occurs far enough in the future that it has a relatively small PV.

If the interest rate were a lot lower it would make sense to go ahead. For example, if the interest rate were 5% the payoff from success would be $107 million (the NPV) and the EV would be $44 million.

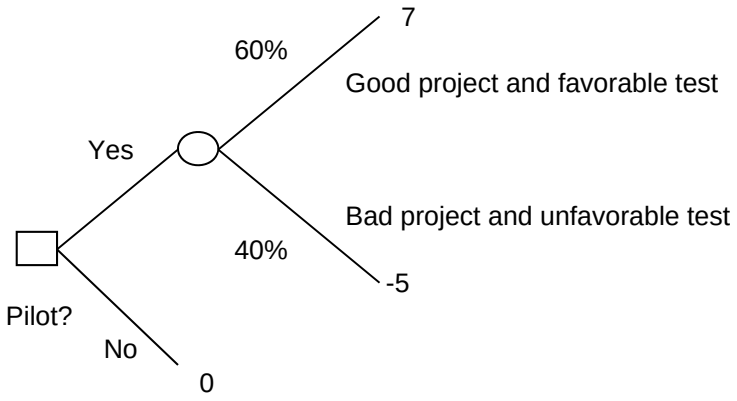


Trimming down the tree:

If the test is favorable, the decision maker will proceed with the research project because doing so will have a net PV of 7.

If the test is unfavorable, the decision maker will not proceed with the research project because doing so would waste an additional 50 million.

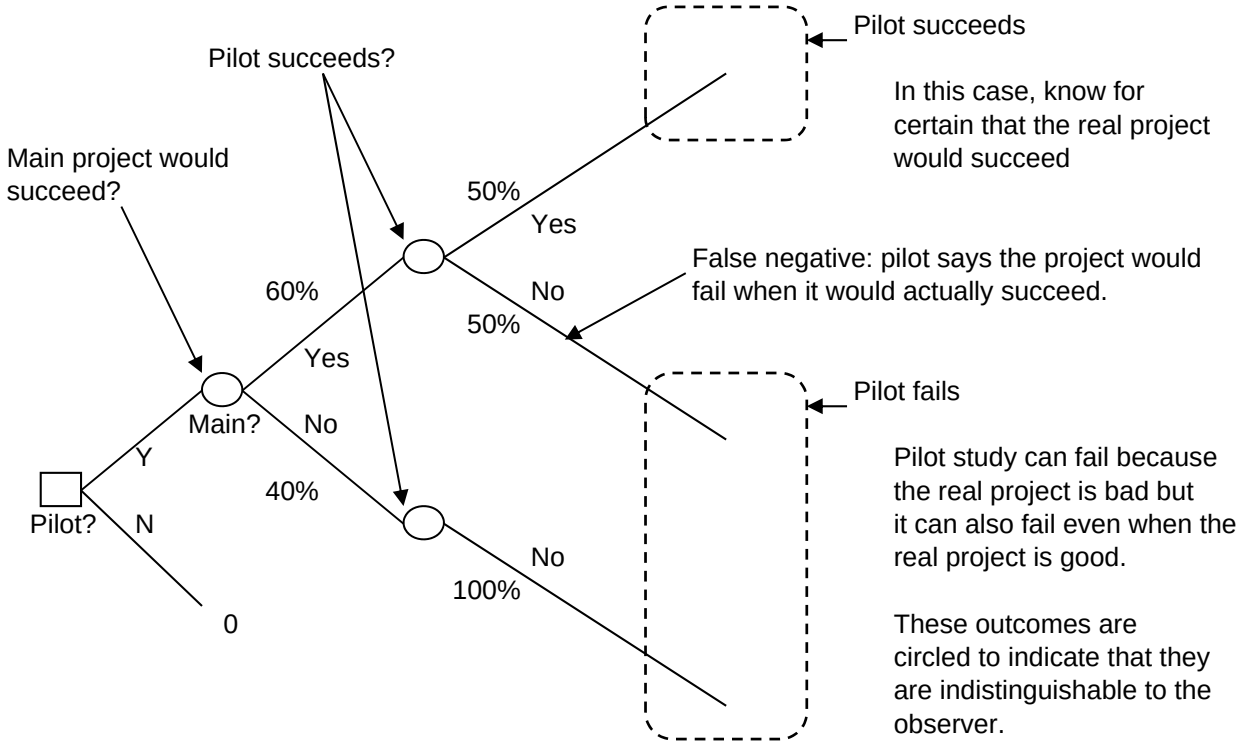
The simplified tree looks like this:



EV of testing = 0.6*7 + 0.4*(-5) = 2.2 million

The firm should order the test.

3 Value of an imperfect test



Given that the pilot fails, what is the probability that the underlying project is actually good?

Probabilities of all outcomes:

Main Good	Pilot Good	Probability
yes	yes	0.6*0.5 = 0.3
yes	no	0.6*0.5 = 0.3
no	yes	0.4*0 = 0
no	no	0.4*1.0 = 0.4

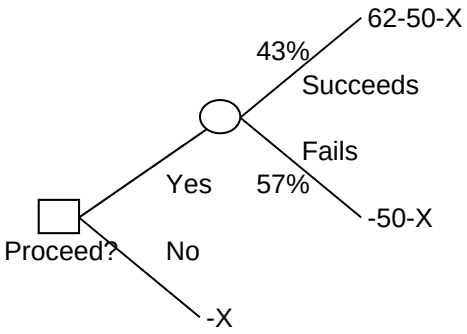
Total chance of a failed pilot study is 0.3 + 0.4 = 0.7

Share of 0.7 from failed pilots for good projects = 0.3/0.7 = .43
Share of 0.7 from failed pilots for bad projects = 0.4/0.7 = .57

Thus, the conditional probabilities for the main project, given that the pilot failed, are:

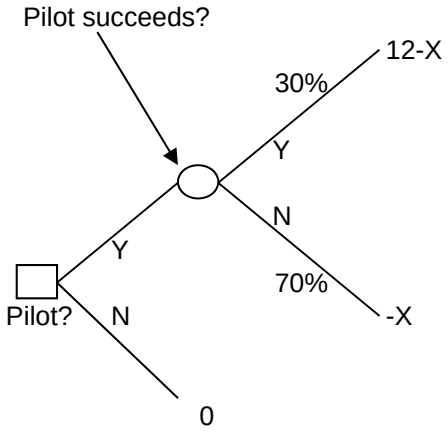
Main project is actually good: 43%
Main project is bad: 57%

If the pilot fails, the firm thus faces the following decision:



EV of proceeding: 0.43*(62-50-X) + 0.57*(-50-X)
EV = 26.6 - 50 - X
EV = -23.4 - X

Since the EV is negative regardless of X, would not want to proceed with the main project if the pilot fails.



Know that the main project is good for certain so the payoff is 12 less the cost of the test, X

Previous step shows that if the pilot fails, the firm shouldn't go ahead with the main project. Payoff is -X, the cost of the test.

EV of the imperfect pilot: 0.3*(12-X) + 0.7*(-X)
EV = 3.6 - X

Better to do the imperfect pilot as long as it costs 3.6 million or less because the EV of the pilot would be positive.