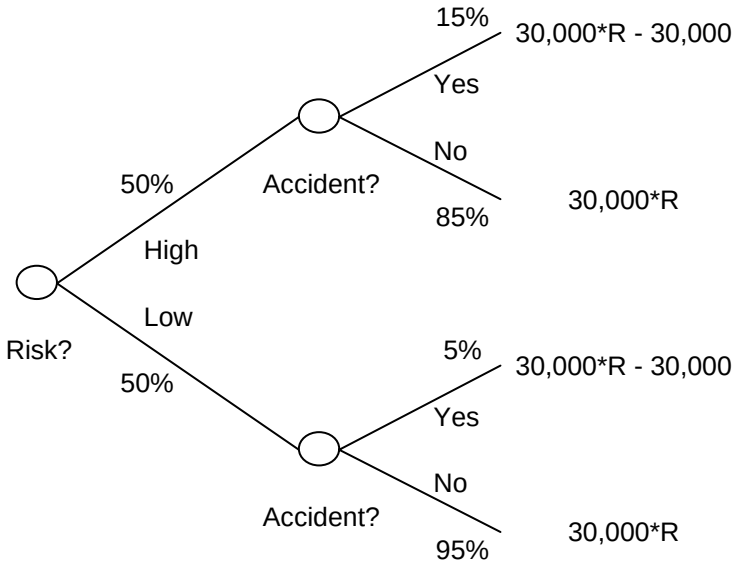
Insurance and pooling of risks
Notes on Solution

1 If all employees are required to buy the policy, then the expected value for the insurance company for any given employee could be summarized in the following tree. Since all employees must buy the insurance, and the firm doesn't know who is who, there's a 50-50 chance that any given employee is high risk.



The insurance company always collects the premium, which is 30,000*R. If the employee is not involved in an accident, that's all that happens. If the employee IS involved in an accident, the company must pay out $30,000.

Expected value of insuring one random employee:

EV = 0.5*(EV of high risk employee) + 0.5*(EV of low risk employee)

EV of high risk = 0.15*(30,000*R - 30,000) + 0.85*(30,000*R)
EV of low risk = 0.05*(30,000*R - 30,000) + 0.95*(30,000*R)

Combining:

EV = 0.5*(0.15*(30,000*R-30,000) + 0.85*(30,000*R))
+ 0.5*(0.05*(30,000*R-30,000) + 0.95*(30,000*R))

EV = 30,000*R - 0.1*30,000 <-- equation showing the EV as a function of R

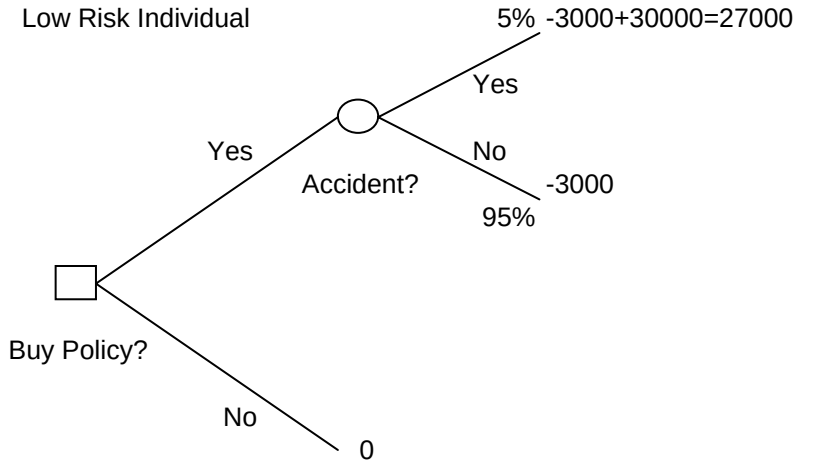
Policy would be fair if R were set so that the EV was zero.
Solving for the R that makes that happen:

0 = 30,000*R - 0.1*30,000
30,000*R = 0.1*30,000
R = 0.1

The premium per dollar of coverage should be equal to the average probability of an accident.

2 Optional insurance but with R=0.1

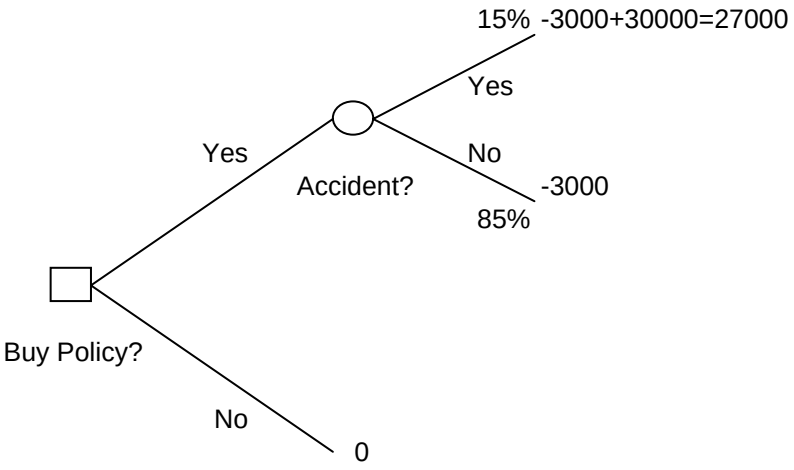
For either individual, the cost of the policy will be 0.1*30,000 = 3,000



EV of buying the policy: 0.05*(27,000) + 0.95*(-3,000) = -1,500
EV of not buying the policy: 0

Low risk person would not buy the policy. Reason: policy would be fair if the person's risk of an accident were 10% but their true risk is only 5%.

High Risk Individual



EV of buying the policy: 0.15*(27,000) + 0.85*(-3,000) = 1,500
EV of not buying the policy: 0

High risk person would buy the policy. Reason: policy would be fair if the person's risk of an accident were 10% but their true risk is actually 15% so the policy has a positive EV.

The implication of this is that a voluntary insurance program would collapse until only high-risk employees participated. As low-risk employees left the program, the probability that any given participant was high-risk would rise until it reached 100% when the last low-risk participant left the program. The insurance company would be losing money and would ultimately have to raise R to 0.15 per dollar of coverage. High-risk employees would end up insured but low-risk employees would have no coverage at all.