

D: Estimation and Monte Carlo analysis

Linking themes of the course:

Measuring and reporting the uncertainty in policy results

Example policy:

Carbon tax on fossil fuels used for electricity generation:

$$T = \$50 \text{ per metric ton of CO}_2$$

Applies to coal and gas

Example model:

Natural gas demand equation in logs:

$$\ln(Q_g) = \beta_0 + \beta_1 \ln(P_g) + \beta_2 \ln(P_c) + \beta_3 \ln(GDP) + \epsilon$$

where:

Variable	Description	Units
Q_g	Natural gas demand	mmBTU (Million BTU)
P_g	Real price of natural gas	2012 dollars/mmBTU
P_c	Real price of coal	2012 dollars/mmBTU
GDP	Real GDP	2012 dollars

Parameter interpretation especially convenient:

Can show:

$$\frac{\partial \ln(Y)}{\partial \ln(X)} \approx \frac{\% \Delta Y}{\% \Delta X}$$

Thus parameters are elasticities:

Own price elasticity:

$$\beta_1 = \frac{\partial \ln(Q_g)}{\partial \ln(P_g)} \approx \frac{\% \Delta Q_g}{\% \Delta P_g}$$

Cross-price elasticity:

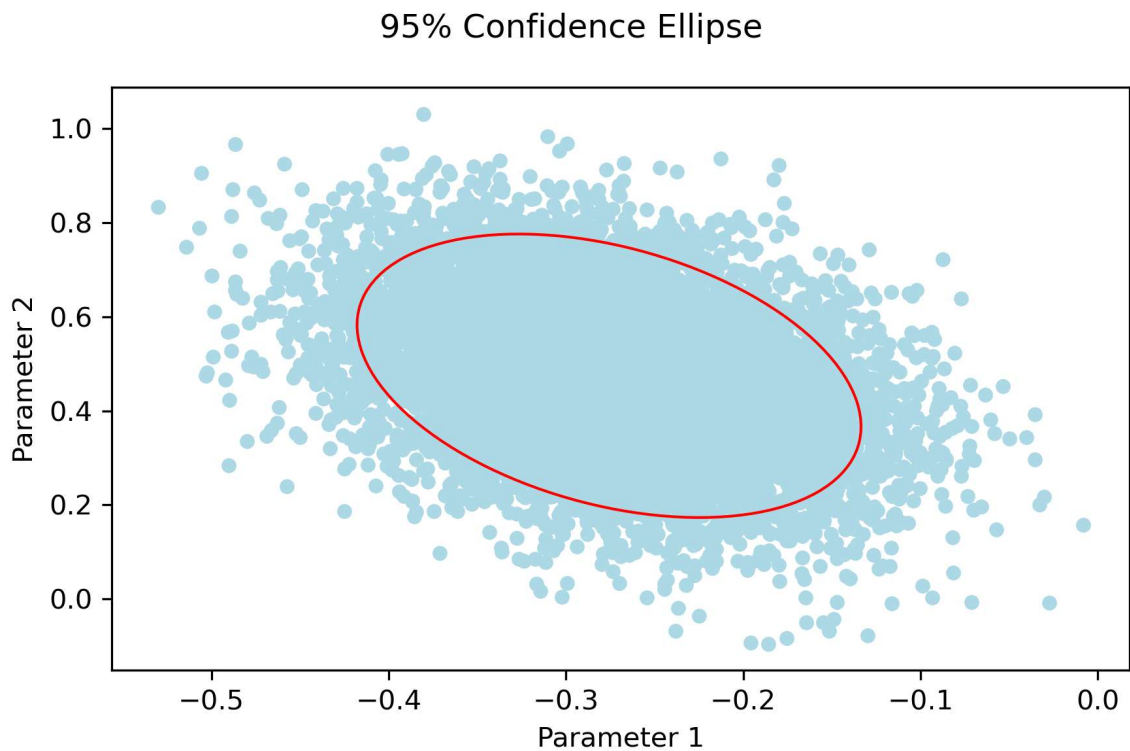
$$\beta_2 = \frac{\partial \ln(Q_g)}{\partial \ln(P_c)} \approx \frac{\% \Delta Q_g}{\% \Delta P_c}$$

Estimated parameters are uncertain and correlated:

Parameter covariance matrix:

$$\Sigma = \begin{bmatrix} \text{var}(\beta_0) & \text{cov}(\beta_0, \beta_1) & \text{cov}(\beta_0, \beta_2) & \dots \\ \text{cov}(\beta_0, \beta_1) & \text{var}(\beta_1) & \dots & \dots \\ \text{cov}(\beta_0, \beta_2) & \dots & \text{var}(\beta_2) & \dots \\ \dots & \dots & \dots & \text{var}(\beta_3) \end{bmatrix}$$

Can show relationships between parameters via confidence ellipses:



Input data for estimation:

EIA Monthly Energy Review, 1973-2022

Key policy concern:

Revenue from CO2 from natural gas:

$$Rev = T * C_g * Q_g$$

where:

T Tax in dollars/metric ton CO2

C_g Tons of CO2 emitted per mmBTU of gas used

Q_g Quantity of gas in mmBTU

Questions:

- Expected tax revenue?
- 90% confidence interval given parameter uncertainty?

See g33 demo.py