E: Efficient incentive design, part 3

Potential gross efficiency gain from biofuels startup:

$108k

Two participants:

Founder (**F**)
Venture capitalist (**VC**)

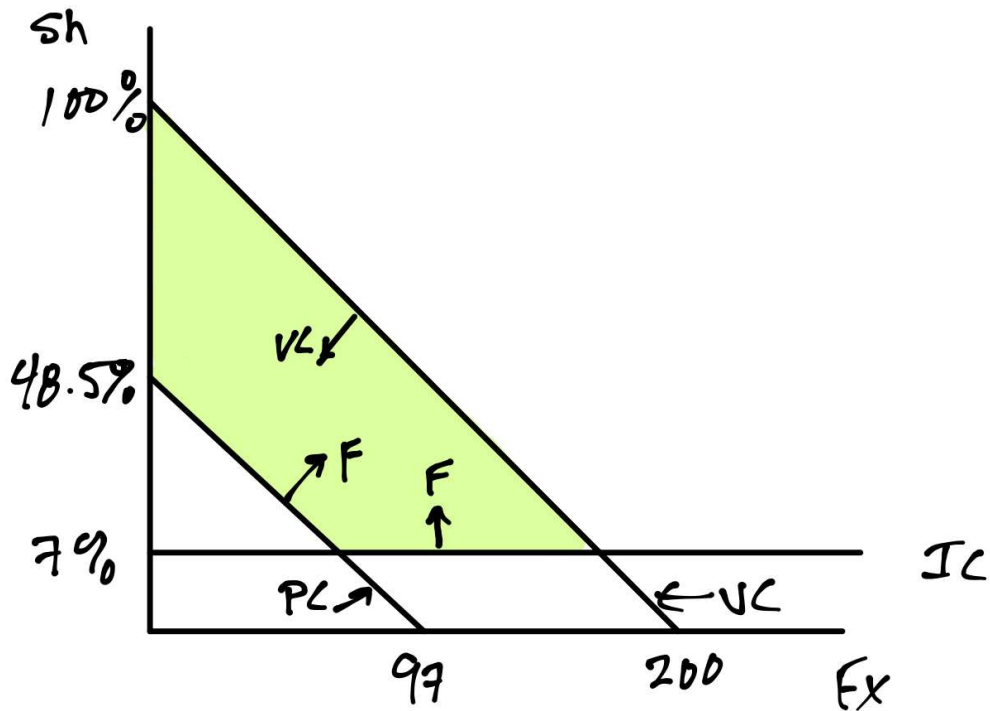Founder's effort is costly and determines probability of success:

Level of effort	Cost to Founder	Prob of Success
High (H):	$5k	20%
Low (L):	$2k	15%

Contract with two parameters:

Fixed payment: **Fx**   VC pays to F
Share of ownership: **Sh**   Retained by F

Range of possible contracts if founder is *risk neutral*:

Adding risk aversion:

Founder's ex post utility from receiving c dollars:

$$u = c^{0.5}$$

Updating the **incentive compatibility** constraint (agent chooses high effort):

$EU_H \geq EU_L$ (EV constraint becomes an EU constraint)

$$EU_H = 0.2 * (Fx + 1000 * Sh - 5)^{0.5} + 0.8 * (Fx + 10 - 5)^{0.5}$$

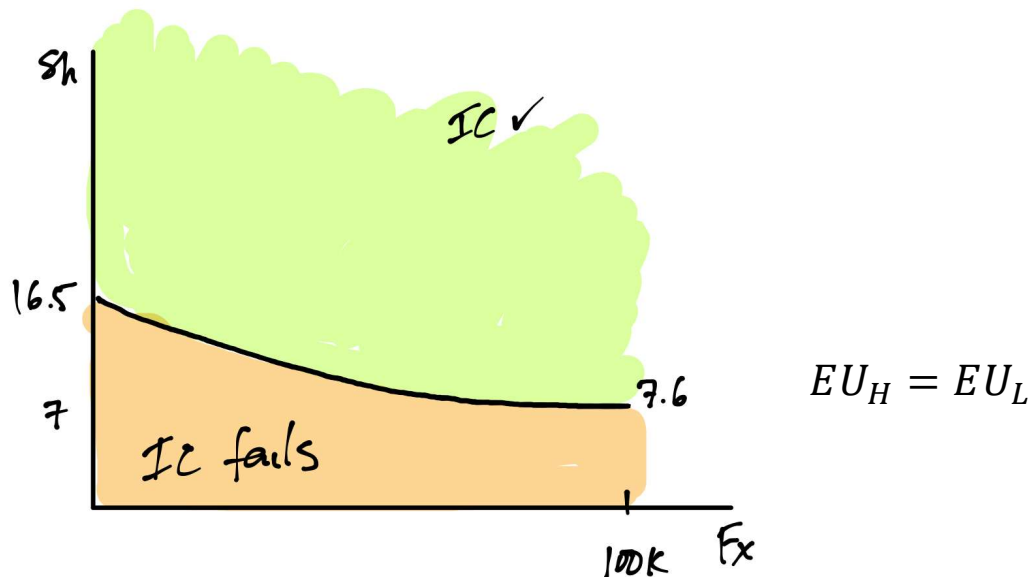$$EV_L = 0.15 * (Fx + 1000 * Sh - 2)^{0.5} + 0.85 * (Fx + 10 - 2)^{0.5}$$

Find combinations of Fx and Sh that solve:

$$0.2 * (Fx + 1000 * Sh - 5)^{0.5} + 0.8 * (Fx + 10 - 5)^{0.5}$$
$$= 0.15 * (Fx + 1000 * Sh - 2)^{0.5} + 0.85 * (Fx + 10 - 2)^{0.5}$$

Hard to do by pencil and paper but easy to compute numerically

Graphing:

$EU_H = EU_L$

Minimum Sh is larger than when risk neutral:

Why?
L effort has more certain outcomes (lower risk) than H
Need to make H relatively more attractive

Participation constraint (agent agrees to the contract):

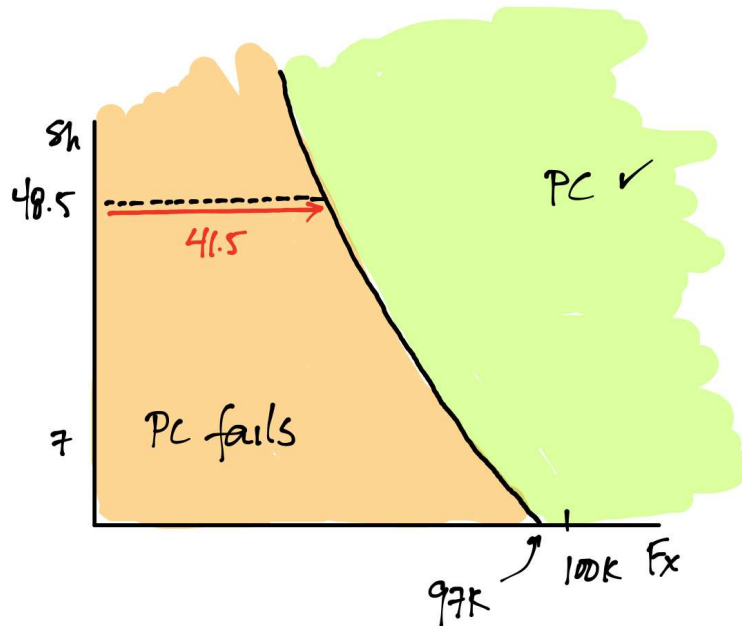$EU_H \geq EU_N$ (EV constraint become an EU constraint)

$EU_H = 0.2 * (Fx + 1000 * Sh - 5)^{0.5} + 0.8 * (Fx + 10 - 5)^{0.5}$

$EU_N = (100)^{0.5}$

Find combinations that solve:

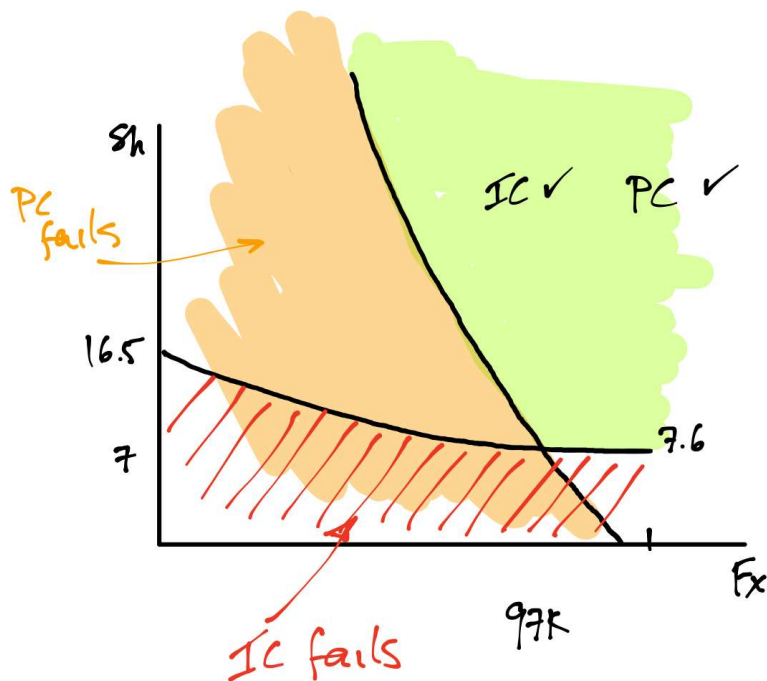$$0.2 * (Fx + 1000 * Sh - 5)^{0.5} + 0.8 * (Fx + 10 - 5)^{0.5} = (100)^{0.5}$$

Graphing:

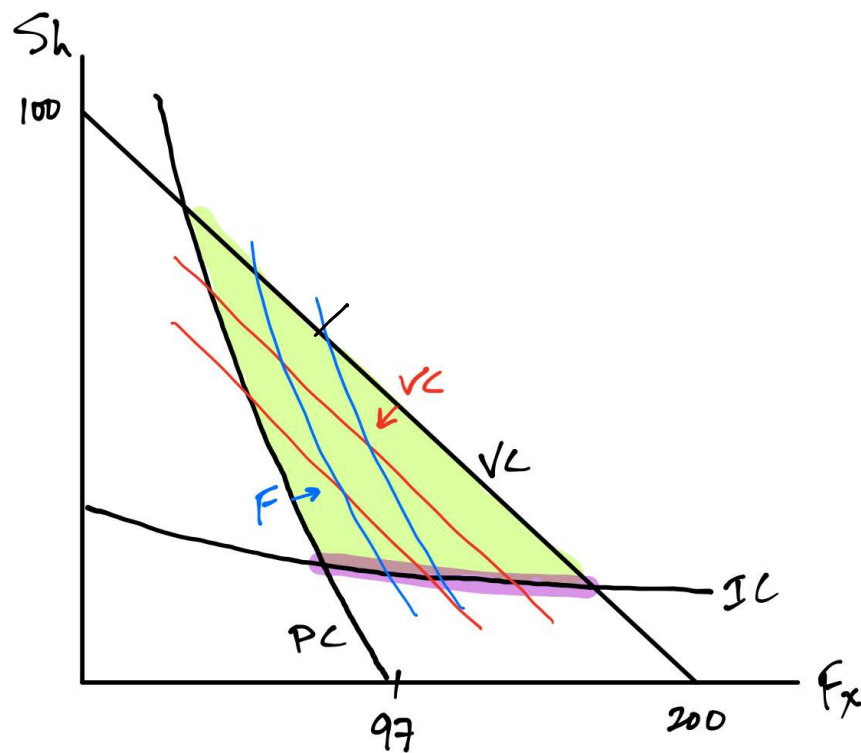**Minimum Fx is much larger for a 48.5%** share (was 0)

Why?
Need to compensate for risk

Combining the founder's constraints:

Sh axis, **Fx** axis

PC fails (orange region)

IC ✓   PC ✓

16.5

7.6

7

97k

IC fails

Adding the VC's constraint (unchanged):

Sh axis

100

VC

VC

F →

IC

PC

97

200

Fx

F prefers:
  Up, Right

VC prefers:
  Down, Left

Efficient:
  Shaded line

For reference, intersection of IC, PC:
$$Sh = 7.7\%, Fx = 83.1k$$

Overall:
- Can solve the PA problem with appropriate contract (incentive) design
- Many contracts possible but differ in who gets the surplus
- Risk aversion changes contract space slightly