

## E: Repeated tests

With imperfect tests, repeated testing can be valuable

- Can reduce uncertainty substantially

Example: potentially risky medical procedure (e.g., gene therapy)

- Patient has severely debilitating medical condition, e.g., ALS
- Experimental treatment with two outcomes, good G and bad B:

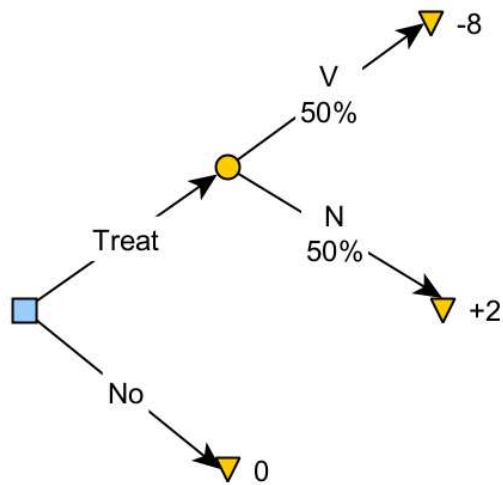
| Outcome                     | Payoff |
|-----------------------------|--------|
| G: Improves quality of life | \$2 M  |
| B: Immediately fatal        | -\$8 M |

- Outcome depends on characteristic of patient (e.g., genetics)

| State                                                | Probability |
|------------------------------------------------------|-------------|
| Vulnerable to treatment (V), e.g., allergic reaction | 50%         |
| Not vulnerable (N)                                   | 50%         |

- Treating a vulnerable person causes the bad outcome

Treat without testing vulnerability:



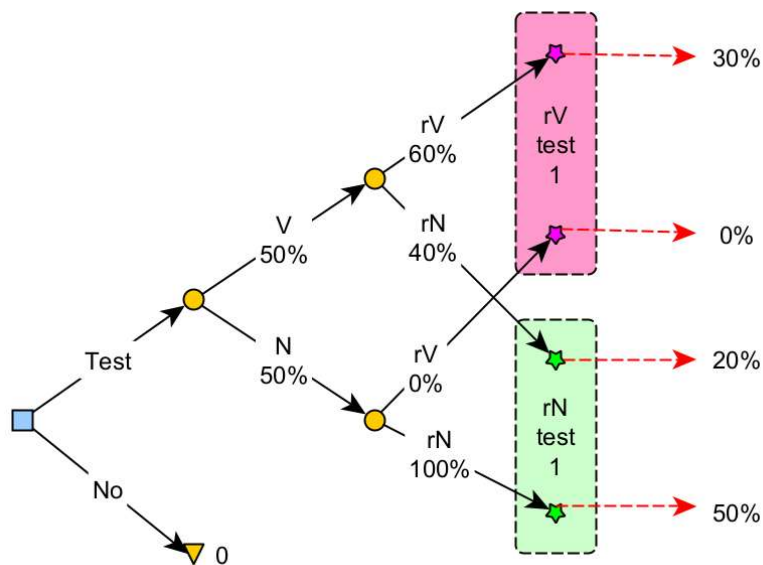
$$EV = 0.5*(-8) + 0.5*2 = -3$$

### Adding a test:

Test performance:

40% chance of a false negative: sometimes rN when true condition is V

0% chance of a false positive: never rV when true condition is N



Stop if rV:

$$\text{Probability of V given rV} = 30\% / (30\% + 0\%) = 100\%$$

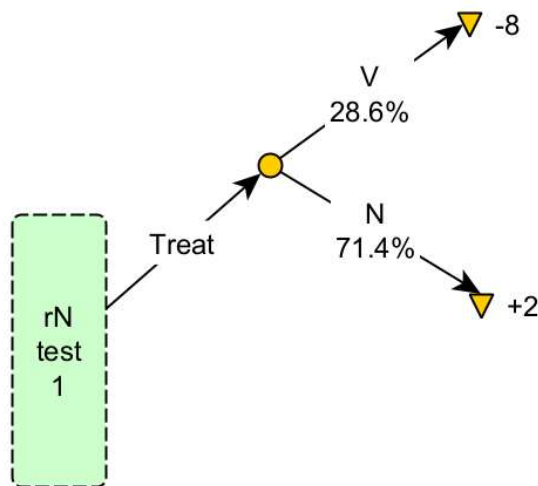
Question: treat if rN?

Step 1: revised probability of V

$$\Pr(rN) = 20\% + 50\% = 70\%$$

$$\Pr(V|rN) = 20\%/70\% = 28.6\%$$

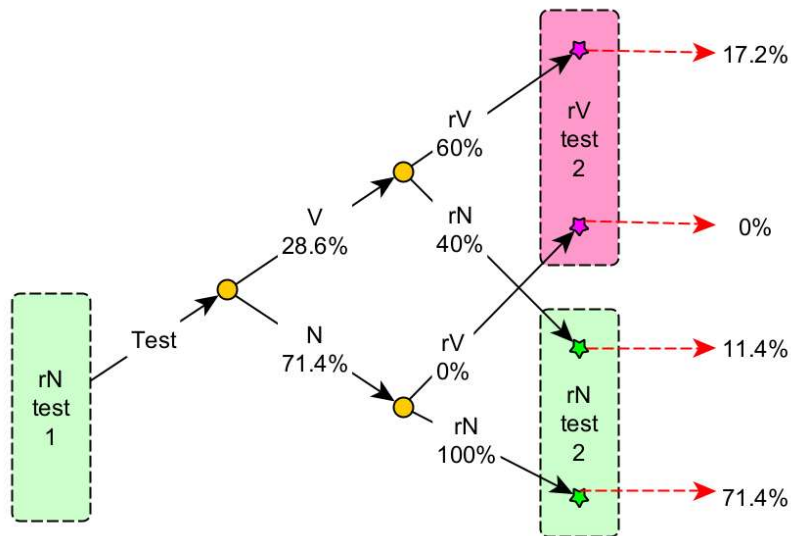
Step 2: decision to treat



$$EV = 0.286*(-8) + 0.714*2 = -0.86$$

Getting a second opinion: retesting if first test is rN

Extend tree beyond the test-1 rN information set:



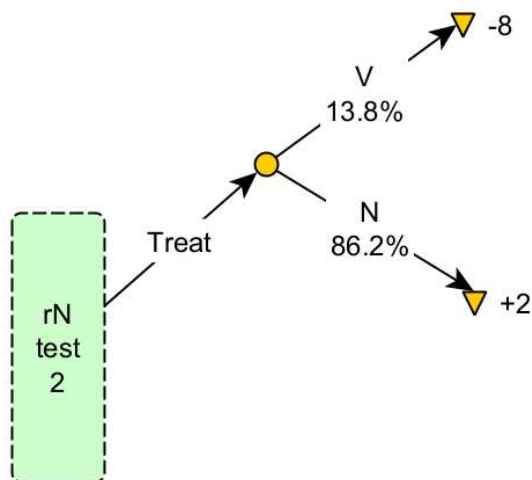
Treat if second rN?

Step 1: revised probability of V

$$\Pr(rN) = 11.4\% + 71.4\% = 82.8\%$$

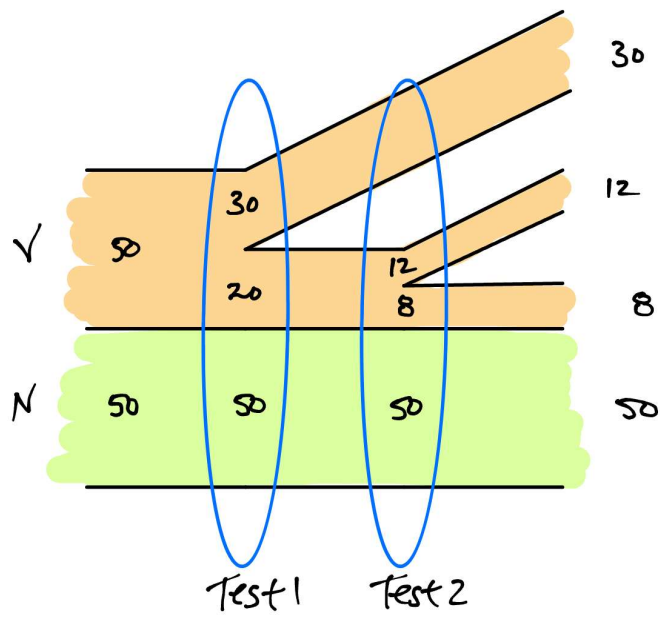
$$\Pr(V|rN) = 11.4\%/82.8\% = 13.8\%$$

Step 2: decision to treat



$$EV = 0.138 * (-8) + 0.862 * 2 = 0.62$$

Intuition: repeated tests filter V people out of the pool:



Number of V people in pool after N tests:  $50 * 0.4^N$